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THE SYNERGIC ROLE OF CUSTOMERS IN OPERATIONS MANAGEMENT

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To Rachele, my loving wife.

Without her, this dissertation would have simply not happened.

Abstract

The advent of Web 2.0 applications in the last ten years has profoundly changed the way people communicate and interact with each other via the internet. Blogs, forums and social networks have expanded the possibilities for people to exchange information; smartphones and tablets have expanded the accessibility of such information. Beyond their undeniable influence on social activity, these changes have also introduced important changes on economic activity, especially in the way that companies think of–and deal with–their customers. An increasing number of players have now come to realize that a truly efficient Supply Chain is one that considers customers as an active part of it, and Web 2.0 applications make this not only possible, but also easy and inexpensive. Inspired by the innovative approach of startups like MyFab and by the trailblazing growth of startups like Groupon, my doctoral thesis uses dynamic games to examine a series of innovative business models where firms use the internet to engage with their customer base and, by doing so, acquire relevant information that they use to improve their business decisions, such as product development, seasonal opening/closing, and pricing. My work also aims at providing recommendations for the correct design of these novel business models, in order to induce truthful voluntary information sharing on the part of customers and maximize the benefit that the firm derives from the acquired information.

The first chapter of my dissertation, "Information Acquisition Through Customer Voting Systems", co-authored with Prof. Karan Girotra, studies the use of customer-centric internet polls on the part of a firm to improve development and pricing decisions. In these systems, customers are presented with a product design, and they can signal their preferences by casting a vote in favor of it. Doing so typically results in some benefit, such as a discount on the future price of the product. Our analysis shows that, depending on the decision to be advised, the firm and the customers may have aligned or conflicting incentives, so that the customers may not be willing to reveal information when it could be used against their interest—as is the case with pricing. We therefore develop two advanced voting systems that tweak the incentive of the parties and allow the firm to acquire information even when used for pricing decisions.

The second and third chapters of my dissertation take a novel operational perspective on an innovative discount structure, pioneered by Groupon and copied by many of its competitors in the daily deal industry, in which a discounted deal is considered valid only if a pre-announced number of customers show interest in the offering–we call these threshold discounting offers. Specifically,

in the second chapter, “Operational Advantages and Optimal Design of Threshold Discounting Offers”, co-authored with Prof. Karan Girotra and Prof. Serguei Netessine, I consider a capacity-constrained service provider who offers his services on different time periods to a random-sized population of strategic customers with heterogeneous service-time preferences. Demand seasonality and inter-temporal demand substitution then arise endogenously in the model. I find that threshold discounting offers outperform traditional demand manipulation approaches by boosting capacity utilization and profit. Interestingly, the improvement persists even in the absence of economies of scale, typically considered as a necessary requirement for group-discounting practices to yield any benefit. Further, by communicating to customers whether the discount is activated or not, the firm signals them the demand state, inducing a strategic response that further improves capacity utilization. Hence, in contrast with the main message from the literature on strategic customers, we show that in our context strategic customers are beneficial to the firm. We then evaluate alternative design options of Threshold Discounting offers and provide recommendations on how to maximize their benefits.

The third chapter of the dissertation, “Threshold Discounting Offers: Unintended Consequences and Incentive Conflicts”, co-authored with Prof. Karan Girotra and Prof. Serguei Netessine, further expand the analysis on threshold discounting offers in several directions. Specifically, we consider cases in which discounts are not just an effective way to smooth demand across time, but are also an effective way to expand the market for a service. Within this context, we show that threshold discounting outperforms traditional approaches for businesses that experience strong seasonality together with sufficient variability in demand, but it may deliver little value beyond traditional approaches when demand smoothing is not a priority for a firm, and is even harmful in situations in which capacity shortages are rare. Since it relies on acquiring information from customers, we show that threshold discounting has diminished value when customers exhibit significant transaction costs to subscribe to discounted deals, making it more costly for the firm to get them involved. Interesting, our analysis is the first attempt to provide an explanation for why the much-celebrated threshold discounting offers were unexpectedly discontinued by Groupon. A first explanation is based on a *lack of fit*: we show that demand-starved firms, which arguably constitute a big proportion of the service providers featured in daily deals websites, derive no value from a threshold discounting offer, since the main advantage of these offers is to better match supply and demand, while such firms have too much supply relative to demand. A second explanation builds on *incentive conflicts in the* *supply chain* that arise when these offers are channeled through an intermediary, as it is common in the industry: in these cases, the intermediary has strong incentives to prefer a deal that has a higher discount and a much lower threshold compared to what would be best for the firm–and many intermediaries are powerful enough to impose their terms on those service providers that want to be featured in their websites. Overall, our results complement our analysis in chapter two by providing elements that caution towards the use of threshold discounting in certain settings, and by using our findings to explain what has happened in practice.

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Contents

Part 1. Information Acquisition Through Customer Voting Systems

We study the use of customer voting systems that enable information acquisition from strategic customers to improve pricing and product development decisions. In these systems, the firm presents customers with a product design and gives them the opportunity to cast a vote on this design, a vote that has costs and benefits. For example, voting may be cumbersome, but those that vote in favor of a design may be eligible for a discount if and when the design gets developed. Customers vote and the firm interprets the voting outcome to discern customer interest in the product, and to advise on further development and/or pricing of the product. We model the interactions between the firm and strategic customers in such systems as a game of incomplete information with voting embedded as a subgame. Our analysis shows that the design and effectiveness of a voting system depends crucially on the intended use of the acquired information. When the acquired information is used to advise on development decisions, where firm and customer interests are aligned, voting systems that reward voters with discounts on subsequent purchase of products, in effect incentivizing voting *in favor* of products, can elicit information from customers and improve profit. On the other hand, when the information is used to set prices, a decision where firm and customer interests are misaligned, such systems are ineffective. In these cases, voting systems that effectively incentivize customers to vote *against* products or those that partially limit the firm's future price flexibility should instead be used to acquire information. While both solutions improve firm profit, the former is preferred for high-value products, while the latter is preferred when voting involves less effort. Based on data for two representative products in the home decor industry, we find that these systems may increase gross product profits by up to 50% for development and by 20-30% for pricing.

1. Introduction

Web 2.0 technologies, social networks, micro-blogs and location-based services have enabled firms to increasingly involve their customer base in business decisions. Such engagement, often referred to as crowdsourcing, typically involves leveraging customer opinion and resources to improve business processes that were traditionally performed opaquely to customers. Such engagement has displaced traditional business models in some industries (cf. Wikipedia (Mikolaj et al. (2012))), while creating new competing business models for product design, research and development, and problem solving (we refer the reader respectively to Threadless (Brabham (2010)), Innocentive (Lakhani (2008)) and Hypios (Girotra and Terwiesch (2010))). This paper introduces a new business process, customer voting systems, through which firms can engage customers in a firm's operational decisions, specifically soliciting their input to improve product development and pricing decisions.

Voting systems were first prominently used by Threadless, where, in addition to designing new products, the community could also vote for designs. Home decor retailer MyFab (Girotra and Netessine (2011); Volongo and Girotra (2012)) refined voting systems by offering purchasing discounts to voters with the explicit purpose of acquiring customer information. Founded in 2008, MyFab had years of trailblazing growth, received over US$10 million in venture financing, and expanded to new markets.[1] Today, these voting systems are employed by firms in other industries like apparel and home decor retail.[2]

At an online retailer using a customer voting system, web visitors are presented with potential product designs. Product specifications, detailed pictures, and in some cases, pricing information are provided. Customers have the opportunity to cast a vote on the product design. Casting a vote comes with costs and benefits. The most thoughtful implementations of voting systems impose some barriers to casting a vote, such as identity verification, email confirmation of vote, etc. At the same time, visitors that complete a vote are offered benefits, for example MyFab offers a 10% discount to all customers that vote for a design and then buy the product if and when it is offered.[3] After customer votes are tallied, the firm uses the data to advise on pricing of the product and/or

[1]For more details on MyFab's business model, see "Furniture Shopping with the Crowds", Springwise, December, 16, 2008, http://goo.gl/AKkrb, "France's MyFab launches in US", TechCrunch, February 12, 2010, http://goo.gl/6dSKC
[2]See "Garmz wants to be the fashion MyFab", TechCrunch, May 12, 2011, http://goo.gl/u1WAj, "How ModCloth's Be-The-Buyer Program Crowdsourced Its Way To Success", Inc., March 10, 2011 http://goo.gl/h8hf and "Made.com raises £2.5m to assault designer home decor industry", TechCrunch, March 21, 2010, http://goo.gl/AeFvv.
[3]"Mass customizing high style, low cost home decor", Cooltownstudios, December 18, 2009 http://goo.gl/guOjy.

on further development of the product. Finally, customers have the opportunity to buy the product if it is developed.

Despite the growing use of customer voting systems and their celebration in the popular press, these systems have not been rigorously studied. In particular, to the best of our knowledge there is no systematic analysis of the relationship between the design of a voting system, the intended use of acquired information, and the system's effectiveness in improving firm profits. While there is a rich operations literature on information sharing within the supply chain, acquiring information from strategic customers, as customer voting systems do, has largely been unaddressed. In the absence of rigorous analysis, practicing firms do not know when and how these voting systems are effective and they experiment with different systems by changing their voting design frequently. Further, even when customers are engaged by the voting system, practicing firms have too limited an understanding of the drivers of customer behavior to use gathered information intelligently (Volongo and Girotra (2012)).

This study builds a stylized model of firm and strategic customer incentives in voting systems to provide guidelines on their design and use. We model firm and strategic customer interaction as a dynamic game of incomplete information. We characterize equilibria where each player's strategies yield a Bayesian Nash Equilibrium in every continuation game given Bayesian posterior beliefs. We compute the value and costs of information acquisition, evaluate the system's effectiveness, and understand the relative merits of different systems.

Our analysis illustrates that the design and effectiveness of voting systems depends crucially on the intended use of the information gathered from voting. We find that when a firm intends to use the information collected to identify if a product should be further developed, a decision where customer and firm interests are aligned (both want to develop the product when there is high interest and vice versa), voting systems that incentivize voting *in favor* of products are effective at acquiring customer information. On the other hand, when information is used to set prices, a decision where customer and firm interests are misaligned (the firm wants to set high prices when there is high interest, while the customers prefer low prices), systems that incentivize voting in favor of products are ineffective. The firm must instead employ other systems, such as one with reverse voting–systems that incent voting *against* products–to acquire and freely use information.

More specifically, for development decisions (Section 3), our analysis shows that a voting system where customers that voted on a product are eligible for a sufficiently high purchasing discount if and when the product is developed, are effective in acquiring information. Customers vote only when they value the product highly, in effect voting *in favor* of the product. This allows the firm to (partially) interpret customer preferences from voting outcomes. We characterize the benefits of employing such systems and find that these are driven by a fine balance between the information gains and the cost of acquiring information from strategic customers. Specifically, these systems are most beneficial for firms/products when the production, shipment and development costs are higher, but their advantage decreases when cost of casting a vote is higher. Further, we unexpectedly find that higher uncertainty in customer preferences may actually decrease the benefits of acquiring information through voting systems.

The same voting systems with purchasing discounts that are effective for advising development decisions are surprisingly ineffective when it comes to advising pricing decisions (Section 4). Strategic customers see no benefit in voting in favor of products and signaling their information, information that in this case may be used against their interest. This leads to an uninformative voting outcome. We develop two novel voting systems that address the incentive conflicts inherent in using customer information to set prices. In the first system, the firm commits to a maximum price: this incentivizes customers to vote in favor of products, while sacrificing some pricing flexibility. In the second, we propose a reverse voting system that flips the customer incentives to vote, making them better off casting a vote when they *do not* like the product. These systems provide incentives for customers to share information even when it is used to set prices. We compare both systems and find that the latter, where customer incentives are reversed, is preferred for products of high value, whereas the former, where the firm commits to a maximum price, provides control on what information is acquired and is preferred when the cost of casting a vote is low.

Numerical estimates, based on data for products typically sold by online retailers implementing voting systems, indicate that appropriately designed voting systems may improve gross product profits by up to 50% for development, and between 20-30% for pricing decisions, compared to business models without any voting mechanism.

Our paper makes three contributions. First, we develop an analytical framework to study customer voting systems, novel and increasingly prominent systems that use customer engagement technologies to advise on operational decisions. To the best of our knowledge, this is the first analysis of these systems. Second, we extend the supply chain literature on information sharing. While existing research has largely focused on information sharing between firms and partners in the supply chain, we examine information sharing between a firm and its strategically acting customers to improve product development and pricing decisions, thanks to the use of customer-engaging voting systems. Finally, we provide practical guidelines on the design and use of customer voting systems, identifying the appropriate system to be employed in different settings, and provide realistic estimates of their effectiveness when used for typical products.

2. Literature Review

Our work is related to three streams in the operations literature: information acquisition in supply chains, tournaments and crowdsourcing, and strategic customers.

Information Acquisition in Supply Chains: Designing mechanisms that enable information flow within members of a supply chain has been widely studied, typically concerning the sharing of demand forecast information (see Oh and Özer (2012) for recent advances and the references therein for an extensive summary of the literature). While this work largely considers the intended sharing of information, another stream has examined the unintended sharing or leakage of information. Li (2002) and Li and Zhang (2008) study the retailer incentives and information leakage in a two-tier supply chain. More recently, Ha et al. (2011) consider information sharing in competing supply chains. Anand and Goyal (2009) examine the leakage of information through material flows in the supply chain. Finally, while most of this work has taken an analytical game-theoretic approach to studying information sharing, Özer et al. (2011) conduct laboratory experiments to examine the relationship between trust and information sharing. In contrast to this literature, which studies information flows within different tiers of a supply chain, this study examines information flows between the supply chain and its strategically acting customers. In a similar vein, Huang and Van Mieghem (2012) study how tracking customer online clicks may provide information to an online seller. However, while they study indirect monitoring systems (click-tracking) and their main focus is on situations where the objective of the firm is to reduce quantitative demand uncertainty on an existing product to improve inventory decisions, we consider active systems in which customers are presented with incentive rules (such as purchasing discounts) in situations where the objective of the firm is to reduce uncertainty around customer valuation for a new product to improve development and pricing decisions.

Tournaments and Crowdsourcing: Engaging customers or other external parties in firm decisions has been studied extensively in the crowdsourcing and tournaments literature. Among the early works, Ehrenberg and Bognanno (1988) find that the level and structure of prizes in tournaments have a significant impact on participants' performance. More recently, Terwiesch and Xu (2008) provide recommendations on the rules to be employed in a contest depending on the type of problem at hand. Boudreau et al. (2011) provide empirical evidence that the number of contestants is important to determine the quality of the best solution. Like this body of research, we examine crowdsourcing; however, while the above studies examine incentives when external parties are engaged in innovation or problem solving, this study is focused on incentives in information elicitation when a firm deals with its customers. The only study about customer voting systems that we are aware of is Caldentey and Araman (2013), but their study has a very different focus from ours, that is, they determine the optimal duration of the voting phase for the firm, accounting for the trade off between better information and earlier product launch.

Strategic Customers: In recent years there has been strong interest in examining the implications of dealing with forward-looking customers that strategically time their purchases, thus harming firm profits. Cachon and Swinney (2009) study the value of quick response and enhanced design strategies in the presence of strategic customers. Su and Zhang (2008) study a two-tier supply chain selling to strategic customers and identify the benefits of decentralization. Su and Zhang (2009) show how quantity commitment and availability guarantees can mitigate the negative effects of strategic customer behavior when a newsvendor sells to strategic customers. Parlaktürk (2012) studies the value of variety when selling to strategic customers. Boyacı and Özer (2010) analyze advance selling in the presence of risk and loss-averse strategic customers. Li and Zhang (2012) consider pre-ordering and show that using pre-order information to improve availability hurts the firm by reducing its ability to price-discriminate. Like these papers, our study models forward-looking customers. While all these studies consider strategic behavior in the timing of the purchase, this study focuses on the customer's strategic *voting* decision, which must account for different incentive systems put in place by an information-seeking seller.

Our work is also related to the political economics literature on social choice theory (for a current survey see Gaertner (2009), and see Condorcet (1785), Arrow (1963) for foundations). Like our work, this literature considers the design of voting systems, but it departs in the design objective. While political voting systems are designed to elicit relative preferences in order to maximize social welfare, customer voting systems are designed by firms to elicit information from customers about their valuation for a product in order to improve business decisions and maximize profit.

To summarize, we extend the information sharing in supply chains literature by considering active sharing between the firm and customers. We consider a new dimension of customer strategic behavior, when customers vote strategically for products. Finally, our work considers a new form of crowdsourcing, that of crowdsourcing operational firm decisions.

In this section, we consider the use of customer voting systems to advise on product development decisions. In the next section, we consider use of these systems to advise on pricing.

3.1. **Preliminaries.** Consider a firm with an innovative product design. Bringing this product to market requires making additional investments of c_F monetary units, representing the cost of finalizing the design, booking supplier capacity, establishing production capabilities, etc. If these development costs are incurred, the firm sells the product to a market of homogenous strategic customers whose valuation for the product, X, is a continuous random variable with convex support S, differentiable pdf f, cdf F, and survival function $\bar{F} = 1 - F$. While customers observe their valuation, the firm only knows its prior distribution. We assume that each customer purchases at most one unit of the product, and that the ensuing demand is fulfilled at a unit variable cost c, which includes both production and delivery costs. Without loss of generality, we normalize the mass of customers in the market to one.

In the absence of any voting mechanism, the firm makes its development decision to maximize expected profit, computed using the common prior distribution on product valuation, f. The optimal price to sell the developed product P_N^* is the root of $\bar{F}(P_N^*) - f(P_N^*)(P_N^* - c) = 0$.[4] If the maximum expected profit is negative, the firm does not develop the product; otherwise, the firm develops the product and its expected profit is

$$\Pi_N^* = \int_{P_N^*}^{\infty} (P_N^* - c)\, dF - c_F, \tag{3.1}$$

where the subscript N is used to identify the metrics for a no-voting business model.

[4] To ensure a unique solution, we require the profit function to be quasi-concave in price, i.e. $\exists! P_N^* : \bar{F}(P_N^*) - f(P_N^*)(P_N^* - c) = 0$, and $-2f(P_N^*) - f'(P_N^*)(P_N^* - c) < 0$. This holds, for example, for valuation distributions with a non-decreasing hazard rate (such as uniform and exponential) and for the normal distribution.

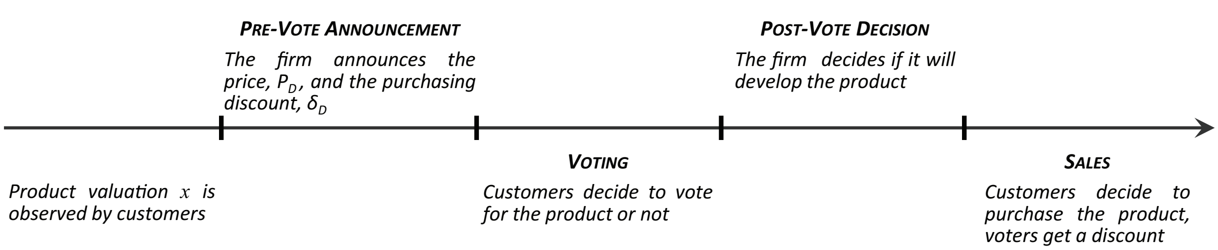


Figure 3.1: Customer Voting System to advise on Development Decisions

3.2. **Customer Voting Systems with Purchasing Discount.** We now consider the use of a customer voting system with purchasing discount at the above described firm. In such a system, the firm puts up a product design for voting and decides to develop the product only after observing the outcome of the poll. The sequence of actions is illustrated in Figure 3.1. First, the product valuation is drawn and the realized valuation x is observed by customers.[5] Next, the firm announces a price P_D, a purchasing discount $\delta_D \leq 1$ (more on this later), and shares the product characteristics (typically pictures and specifications) with its customers. Each customer then chooses whether or not to vote for the product. Casting a vote involves customer effort (for example, verification of identity, email confirmation of vote, using up the limited number of votes, etc.) captured through a cost of casting a vote, $c_v > 0$.[6] Next, the firm uses the observed outcome of the poll to decide whether to incur the development cost c_F. If the product is developed, it is available for sale: customers who previously voted for the product can purchase it at the discounted price $\delta_D P_D$, while customers who did not vote for it must pay the full price P_D. Essentially, customers who vote for the product earn the right to purchase it at a discounted price. The subscript D is used to identify metrics associated with this voting system used to advise on development decisions.

The above sequence of actions constitutes a game of incomplete information with infinitely many players. We search for equilibria where each player's strategies yield a Bayesian Nash Equilibrium in every continuation game given the posterior beliefs of the players, beliefs that are updated in accordance with Bayes' law (Fudenberg and Tirole (1991), page 321). As is typical in the analysis of such games, it is convenient to describe the optimal strategies of each player in reverse chronological order. For any price P_D and discount δ_D, the buying strategy for any customer is to buy the product iff she makes a positive surplus from the trade, that is, iff $x - \delta_D P_D \geq 0$ for customers that voted for the product, or iff $x - P_D \geq 0$ for customers that did not vote for the product. This is preceded by the firm's development decision, where based on the voting outcome, that is the fraction of customers, ν_o, that cast a vote in favor of the product, the firm updates its prior information f on customer valuation for the product to the posterior information f_{ν_o}. The firm then develops

[5] Voting systems are typically used for new products, where the principal source of uncertainty for the firm is whether the product is met favorably by the market as a whole, and idiosyncratic differences within the market, i.e. between individual customers, are a secondary concern. Considering customers with homogenous yet uncertain valuations for the product allows us to focus on the first type of uncertainty, i.e. on the *aggregate* market uncertainty faced by the firm, which is the more relevant to our context. We refer the reader to **??** for a discussion on potential extensions to the model.

[6] See Hann and Terwiesch (2003) for estimates on the costs of online customer actions.

the product iff, accounting for such updated information, the expected profit-to-go $\pi_D^{\nu_0}(\delta_D, P_D)$ is positive. In order to characterize f_{ν_o} and $\pi_D^{\nu_0}(\delta_D, P_D)$, we examine the voting step.

We model voting as a simultaneous-move (sub)game among customers. This simultaneous-move voting subgame belongs to a class of games known as coordination games, first defined by Schelling (1960). As is typical in the analysis of such games, we use Harsanyi and Selten's well-known concept of payoff dominance (Harsanyi and Selten (1988), p.81) to identify the equilibrium that arises in the voting step subgame. In our context, this implies that the equilibrium characterized by the voting threshold that maximizes customer surplus will arise. The decision of customer i, after observing the firm's choice of δ_D and P_D and the realized valuation x, can generally be described as casting a vote iff her valuation belongs to a set V_D^i. Her voting strategy is then defined by the set function $V_D^i(\delta_D, P_D)$, and it must be the best response to other customers' voting strategies $V_D^{-i}(\delta_D, P_D)$, taking into account the firm's development strategy and the customer's buying strategy outlined above. We show that in equilibrium all customers vote according to the same voting strategy, and that such an equilibrium strategy is of a threshold type where customers cast a vote iff $x \geq \bar{x}_D^*(\delta_D, P_D)$ (Lemma 5, Appendix A). Casting a vote is an inconvenience for customers, but they may be willing to overcome such inconvenience and vote if they value the product highly enough, due to the benefit of earning a purchasing discount.

At the beginning of the game, anticipating how customer voting strategy $\bar{x}_D^*(\delta_D, P_D)$ responds to its decisions, the firm announces a price P_D^* and a discount δ_D^* that maximize the weighted sum of the profits-to-go for each of the possible voting outcomes, with $\pi_D^{\nu_0}(\delta_D, P_D) = \left(\delta_D^{\nu_o} P_D - c\right) \cdot \bar{F}_{v_o}\left(\delta_D^{\nu_o} P_D\right) - c_F$ being the firm profit-to-go once the voting outcome ν_o is observed, $\bar{F}_{v_o}$ is the survival function of the posterior information obtained using Bayes rule, and where $\delta_D^{\nu_o} = 1 - (1 - \delta_D)\,\nu_o$. If the voting system does not elicit any information, it is no better than a business model without voting. Thus, we require the firm to choose an initial announcement (δ_D, P_D) such that an informative equilibrium exists, that is, where $F\left(\bar{x}_D^*(\delta_D, P_D)\right) \in (0, 1)$, so that different voting outcomes may arise. The next Lemma describes the equilibrium outcome when a voting system is deployed to advise on development decisions. We assume that at least in the best-case scenario, the trade surplus will be enough to compensate for total costs, i.e. $sup\,(S) > c_F + c + c_v$, to avoid the trivial case where it is always better not to develop the product.

Lemma 1. *In voting systems with purchasing discount, there exist informative equilibria only if $\delta_D < 1$. All such informative equilibria have the same profit, and are characterized by a price P_D^*,* *a discount δ_D^*, and a customer voting strategy $\bar{x}_D^*$ such that*

$$\delta_D^* P_D^* = \frac{1 - F\left(\delta_D^* P_D^* + c_v\right)}{f\left(\delta_D^* P_D^* + c_v\right)} + c + c_F \qquad \delta_D^* \leq 1 - \frac{c_v}{P_D^*} \qquad \bar{x}_D^* = \delta_D^* P_D^* + c_v. \tag{3.2}$$

The firm develops the product iff $\nu_o = 1$.

The above Lemma demonstrates that appropriately designed voting systems with purchasing discount are an effective information-elicitation mechanism. For voting to elicit customer information, it is necessary that customers vote only under certain states of the world. This requires a fine balance between the costs and benefits of voting, a balance that tilts differently in different states of the world. Two conditions ensure this: first, voting must come at a cost to customers, if this is not the case, customers would always vote. However, this implies that no customer would incur the cost of voting unless there is some benefit to compensate for it, which leads to the second condition– voting should bring sufficient benefit to customers in *some* states of the world. By offering a high enough discount on the potential purchase of a product, the firm can induce customers to vote for it when they value the product sufficiently highly. In the process, the firm can interpret the voting outcome to acquire improved information on their valuation. Formally, for a sufficiently lucrative discount, $\delta_D^* \leq 1 - \frac{c_v}{P_D^*}$, customers only vote when they value the product highly, $x \geq \delta_D^* P_D^* + c_v$. This allows the firm to update its prior information on customer valuation for the product and eliminate instances where the firm invests in developing a product that is not valued sufficiently by customers.

From a managerial point of view, the result highlights two things. First, that voting systems with purchasing discount may effectively acquire customer information. Second, that only systems that offer discounts to voters provide economic incentives for customers to share information. On the contrary, voting systems like the one used at Threadless, which offers no reward to voters and merely relies on social incentives, may not supply accurate, interpretable information to the firm, given the notorious complexity of social interactions and the firm's limited control on them. Information from such systems should therefore be used with caution.

While voting systems allow a firm to acquire information and potentially increase profits, setting the price and discount so as to elicit information from customers can potentially decrease profits. The next section characterizes this tradeoff and identifies settings where such voting systems are most useful.

3.3. **Advantage of Voting Systems with Purchasing Discount.**

Theorem 1.

1) Voting systems with purchasing discount outperform no-voting business models iff the cost of development is higher than a threshold development cost $\hat{c}_F$, defined by

$$\hat{c}_F = \frac{\bar{F}\left(P_N^*\right)\left(P_N - c\right) - \bar{F}\left(\delta_D^* P_D^* + c_v\right)\left(\delta_D^* P_D^* - c\right)}{F\left(\delta_D^* P_D^* + c_v\right)},$$

where δ_D^* and P_D^* are defined in Equation 3.2 and P_N^* in Section 3.1. This threshold increases in the cost of voting c_v and decreases in the unit cost c.

2) When the firm's uncertainty around customer valuation for the product X increases, the benefit of voting systems, $\Pi_D^* - \Pi_N^*$, can increase or decrease. When X is distributed uniformly over $[b-a, b+a]$, an increase in customer valuation uncertainty, a, decreases the benefit of voting systems iff $b < c + \frac{c_v + c_F}{2}$.

The condition in the first part of Theorem 1 characterizes the circumstances where a voting system outperforms a no-voting business model. This condition can be understood by noting that a voting system differs from a no-voting business model in two principal ways. On the one hand, a voting system allows the firm to halt the development of a product when the voting outcome reveals that customers do not like the product enough. This positive effect, called *loss avoidance*, allows the firm to avoid developing unprofitable products, and it increases in the development cost, c_F. On the other hand, in order to obtain information, voting systems require customers to incur an effort cost to signal their high interest for the product, a cost to customers that does not translate into revenues for the firm, a system inefficiency. Such *voting effort* effect is negative and becomes more prominent as the cost of voting c_v increases. From these two effects follows the existence of the development cost threshold $\hat{c}_F$ above which a voting system is a better choice than a no-voting business model. It also follows that this threshold increases in c_v.

Formally, the benefit of a voting system, $\Pi_D^* - \Pi_N^*$, can be expressed as the sum of the two main effects, *voting effort* and *loss avoidance*, respectively equal to $-(F(P_N^* + c_v) - F(P_N^*))(P_N^* - c)$ and $c_F F(P_N^* + c_v)$, plus a third indirect effect, which captures the different prices charged in the two systems on account of the two main effects.[7] The ability of a voting system to avoid losses and the inefficiency that arises from the voting effort also change the firm's optimal pricing decision.

[7]This component of the profit difference is $c_F\left[F\left(\delta_D^* P_D^* + c_v\right) - F\left(P_N^* + c_v\right)\right] + \bar{F}\left(\delta_D^* P_D^* + c_v\right)\left(\delta_D^* P_D^* - c\right) - \bar{F}\left(P_N^* + c_v\right)\left(P_N^* - c\right)$.

Interestingly, the threshold $\hat{c}_F$ decreases in the unit cost c, meaning that a higher unit cost always increases the advantage of voting systems. This is because a voting system has lower expected sales compared to a no-voting business model (Lemma 6, Appendix A). Both the loss avoidance and the voting effort effects reduce sales, the former because it makes the firm better off charging higher prices, as it commercializes the product only when it is highly valued by customers, and the latter because it shifts the demand curve downward on account of the costs of voting. With lower sales, the negative impact of an increase in unit costs is reduced, making voting systems a more advantageous choice.

The second part of Theorem 1 highlights the interesting (and surprising) role of uncertainty. One expects that higher uncertainty around customer valuation for the product makes information about it more valuable, and consequently the advantages of information-acquiring voting systems should increase with valuation uncertainty. Our analysis shows that this is not always the case when the primary use of information is stopping development in low-valuation states. A mean-preserving increase in the uncertainty around customer valuation for the product implies a fatter right-tail of the valuation distribution. For high-cost products that are profitable only for right-tail valuations, this implies a lower chance of unprofitable states of the world where the voting system's loss avoidance is helpful, thus reducing the benefit of voting systems. Further, this increased tail mass can contribute less to profits for voting systems when the voting system margin is lower. When product valuation is distributed uniformly in the interval $[b-a, b+a]$, the simple condition $b < c + \frac{c_v + c_F}{2}$ characterizes all situations where an increase in market uncertainty reduces the benefit of a voting system. Essentially, when the costs associated with the product (voting, development, and unit cost) are high, a higher uncertainty in customer valuation may operate in the opposite direction of what intuition suggests, thus reducing the benefit of acquiring information through voting systems.

Taken together, our analysis suggests that voting systems with purchasing discount are a helpful innovation to existing business models that can improve firm profits. In particular, voting systems engage customers in firm operations, solicit their inputs on decisions and use these inputs to improve profits. The benefits are most salient for firms/products when the production, shipment and development costs are high. Section 5 uses real data to provide numerical estimates of these gains.

We next consider a voting system that works along the same lines as the ones described in this section, but where the information acquired through voting is used to set prices.

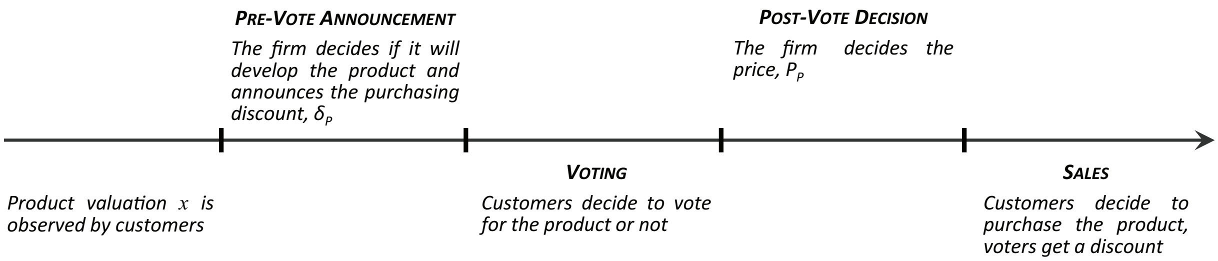


FIGURE 3.2: Customer Voting System to advise on Pricing Decisions

4. Customer Voting Systems to advise on Pricing decisions

4.1. **Customer Voting Systems with Purchasing Discount.** This system follows along the same lines as the voting system to advise on development decisions, except that the decisions of the firm in the pre-vote announcement step and the post-vote decision step are exchanged (Figure 3.2). As before, first the product valuation x is drawn. In the pre-vote step, the firm decides whether to develop the product: if it doesn't, the game ends and it earns zero profit, otherwise it incurs a development cost c_F and then announces a discount δ_P for customers that will vote in favor of the product. In the voting step that follows, after observing the magnitude of the purchasing discount offered by the firm, each customer chooses whether or not to vote in favor of the product. The firm observes the voting outcome ν_o, updates its prior information f to f_{ν_o}, and chooses the price of the product. Finally, customers buy the product– at the reduced price $\delta_P P_P$ if they previously voted for it, or at the full price P_P otherwise.

As before, customers' strategy in the purchasing step is to buy the product iff they make a positive surplus. In the post-vote decision step, the firm pricing strategy $P_P^{\nu_o}(\delta_P)$ maximizes the expected profit-to-go taking into account the discount δ_P announced at the beginning of the game, and bases its decision on the new information f_{ν_o} acquired by observing the voting outcome ν_o. The optimal pricing strategy for the firm is then $P_P^{\nu_o}(\delta_P) = \arg\max_{P_P} \left(\left(\delta_P^{\nu_o} P_P - c \right) \cdot \bar{F}_{\nu_o}\left(\delta_P^{\nu_o} P_P \right) \right)$, where $\delta_P^{\nu_o} = 1 - (1-\delta_P)\,\nu_o$. Customer voting strategy $\bar{x}_P^*(\delta_P)$ is the one that maximizes customer surplus for every announced discount δ_P, taking into account the firm pricing strategy. The firm optimal strategy during the pre-vote step is to choose the discount δ_P that maximizes the weighted sum of the profits-to-go, these being $\pi_P^{\nu_o}(\delta_P) = \left(\delta_P^{\nu_o} P_P^{\nu_o}(\delta_P) - c \right) \cdot \bar{F}_{\nu_o}\left(\delta_P^{\nu_o} P_P^{\nu_o}(\delta_P) \right)$, where the notation is the same as in Section 3. The firm develops the product iff the expected profit above is less than the development cost c_F.

Theorem 2. *There exist no informative equilibria when customer voting systems with purchasing discount are used to advise on pricing decisions.*

Theorem 2 shows that offering customers a purchasing discount does not help the firm acquire information to advise on pricing decisions. This result is in contrast with our previous analysis, where the same voting system was shown to be effective in acquiring information to advise on development decisions. Note that in both these systems the inconvenience of voting, together with the fact that a purchasing discount is most valuable when valuation is high, imply that informative voting may happen only in the high-valuation contingency. However, the incentives of strategic customers to share information in the high valuation contingency depart drastically depending on the intended use of the acquired information.

In a system to advise on *development* decisions, in the high-valuation contingency customers want the product to be developed so that they can make a positive surplus by purchasing it. If the firm finds out about customers' high valuation for the product, it also wants to develop the product, as it is going to be profitable. The customers are better off voting in the high valuation contingency because the firm's self-interested response to their signal is also in their interest, and this drives the effective use of voting systems with purchasing discount. This is not the case with pricing decisions. When the product is valued highly, customers would like the product to be priced *as low as possible* so that they can obtain a higher surplus from purchasing it. But once the firm finds out about the customers' high valuation, it prefers to charge a *higher* price, as customers value the product more. Furthermore, the firm's pricing decision will not compensate customers for having incurred the cost of voting, this being a sunk cost by the time the pricing decision is taken. Thus, when pricing is postponed, customers do not want to share their information because the firm's self-interested response to their signal is counter to their interest.

The above result highlights the importance of considering the intended use of the information in designing voting systems that acquire information from strategic customers. While purchasing discounts are appropriate for advising on development decisions, they are ineffective for advising on pricing decisions. Further, this result is in contrast with the main message of the literature on postponement (see for example Aviv and Federgruen (2001), Biller et al. (2006), and Van Mieghem and Dada (1999)), i.e. that *ceteris paribus*, postponing price or quantity decisions always helps the profits of a monopolist firm. In our work, information is not available as a result of an exogenous process, but is actually acquired by incentivizing customers to share information. This difference is a game-changer: whenever information sharing is an endogenous process, and as such it is conditional on the incentives of the parties being aligned, there is value in postponement only insofar as the decisions being postponed do not subvert the preexisting alignment of interests between the parties.

This inefficacy of voting systems with purchasing discount does not change when the firm uses the acquired information to advise on both development and pricing decisions. In principle, one would expect such a system to perform even better than the previously studied voting system to solely advise on development decisions– the acquired information can be used to advise on two decisions rather than just one. However, as before, the conflict of incentives generated by using information to improve the postponed pricing decision makes customers unwilling to reveal their high valuation, making the information exchange impossible.

From a practical point of view, the results of Theorems 1 and 2 suggest that internet retailers can use voting systems with purchasing discount only when it comes to advising decisions where the interests of the firm and customers in the voting states of the world are aligned. This implies that the use of voting systems to advise on pricing decisions is misguided and likely to lead firms to interpret irrational information, consequently choosing sub-optimal prices. Nevertheless, in many product categories, arriving at the right price for the product is an important strategic objective and there is increasing demand for the design of voting systems that can help advise on pricing decisions. We next exploit our above analysis of strategic customer behavior in voting systems to propose two novel system designs that can be used to advise on pricing decisions, as well as on other decision variables where the incentives of the customers and the firm may not be aligned.

4.2. **Alternate Voting Systems to advise on Pricing decisions.** In our first alternate voting system, the firm commits to restricting the use of information obtained. In the second system, the firm reverses voter incentives by replacing purchasing discounts with penalties, inducing customers to vote against the product, rather than in favor of it. We search for equilibria in threshold voting strategies.

4.2.1. *Voting Systems with Bounded Pricing.* The sequence of actions in this system (illustrated in Figure 4.1) is the same as that in the above voting system to advise on pricing decisions, except for one key difference: the firm's pre-vote announcement now includes, in addition to a purchasing discount δ_B, a binding commitment to a maximum price $\bar{P}_B$ for the product. Hence, the firm must now choose both the discount and the upper bound on price before voting takes place, its optimal

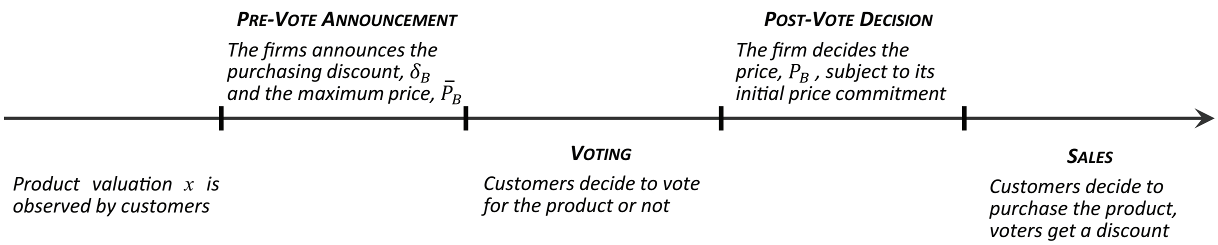


FIGURE 4.1: Customer Voting System with Bounded Pricing

announcement $\left(\delta_B^*, \bar{P}_B^*\right)$ being the one that maximizes $\sum_{\nu_o} Pr\left\{\nu_o|\bar{x}_B^*\right\}\cdot\left(\delta_B^{\nu_o}P_B^{\nu_o *}-c\right)\cdot\bar{F}_{\nu_o}\left(\delta_B^{\nu_o}P_B^{\nu_o *}\right)$, where $\bar{x}_B^*$ is customer equilibrium voting strategy, $P_B^{\nu_o *}=\underset{P_B^{\nu_o}\leq\bar{P}_B}{\arg\max}\left(\delta_B^{\nu_o}P_B^{\nu_o}-c\right)\cdot\bar{F}_{\nu_o}\left(\delta_B^{\nu_o}P_B^{\nu_o}\right)$ are the subgame-perfect pricing functions, notation for $\delta_B^{\nu_o}$ and $\bar{F}_{\nu_o}$ is as before, and where we naturally focus on information-eliciting announcements of the firm. Note that by setting $\left(\delta_B, \bar{P}_B\right)$ the firm affects the future pricing strategy both directly through the upper bound $\bar{P}_B^*$, and indirectly through customer voting strategy $\bar{x}_B^*$, which is a function of the announcement.

As before, in this and in the next voting system that we analyze, we characterize the equilibria where each player's strategies yield a Bayesian Nash Equilibrium in every continuation game given the posterior beliefs of the players, beliefs that are updated in accordance with Bayes' Law, and we use payoff dominance (Harsanyi and Selten (1988), p.81) to identify the equilibrium that arises in the voting step subgame. The equilibrium strategies for this system are provided in Appendix A (page 109). Henceforth, we focus on the interesting case where c_F is low enough for the firm to be better off developing the product: if not, the product is not developed in the first place, hence the firm does not seek to obtain information from customers and the resulting profit is zero.[8] The ensuing equilibrium outcome in a voting system with bounded pricing departs from the one with purchasing discount alone, most interestingly in how information is shared.

Theorem 3. *In a voting system with bounded pricing there always exists an informative equilibrium, i.e. where* $F\left(\bar{x}_B^*\right)\in(0,1)$.

Committing to a maximum price allows the firm to acquire information. As before, when a purchasing discount is offered, customers vote only when the valuation for the product is high enough. But now, unlike before, the firm can commit to not increasing its price to a level where customers would be left with a negative surplus in the high-valuation contingency. This creates incentives for strategic customers to vote and share their private information to extract some surplus. While

[8]Formally, we consider $c_F<\bar{F}\left(\bar{x}_B^*\right)\left(\delta_B^*P_B^{1*}-c\right)+\left[F\left(\bar{x}_B^*\right)-F\left(P_B^{0*}\right)\right]\left(P_B^{0*}-c\right)$.

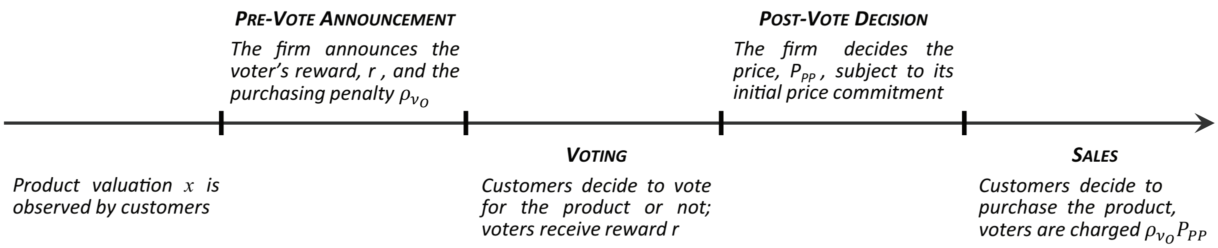


FIGURE 4.2: System with Reverse Voting

committing to a maximum price allows the firm to obtain information, the very same commitment restricts the firm's ability to fully use the information. The next system we propose achieves both objectives– the firm is able to acquire customer information while retaining the flexibility to use the information in the way it sees fit.

4.2.2. *Systems with Reverse Voting.* Inspired by the study of reverse voting systems in political settings,[9] we consider a system that flips the incentives of voters using voter rewards and purchaser penalties. With this mechanism (Figure 4.2), the firm announces that all customers that cast a vote will receive an immediate lump-sum reward $r > 0$ in the form of a micro-payment or a coupon. However, if these customers decide to later purchase the product they may be charged a higher price, a purchasing penalty $\rho_{v_o} \geq 1 \, \forall v_o$ is applied to them. Essentially, voters earn an immediate reward but may also be subject to a price penalty if they buy the product. At this point, each customer decides whether or not to vote. Then, the firm observes the voting outcome ν_o and uses the updated information f_{ν_o} to decide what price P_R to charge for the product. Finally, customers are allowed to buy it– at an augmented price $P_R \cdot \rho_{v_o}$ for voters, and at a regular price P_R for non-voters.

Customers' purchasing strategy is to buy iff their valuation is higher than the price they are charged. The pricing decision of the firm is the solution to $P_R^{\nu_o *} = \arg\max_{P_R} (P_R \cdot \rho_{v_o} - c) \cdot \bar{F}_{\nu_o}(P_R \cdot \rho_{v_o})$ with the usual notation for $\bar{F}_{\nu_o}$. In the voting step, customers vote iff their valuation is *below* a given threshold valuation $\bar{x}_R$. Note that this customer voting strategy is the reversal of that in the other voting systems: customers vote iff their valuation is *below* a given threshold, rather than above. In fact, when the valuation for the product is high, customers prefer to buy it, and since voting for the product in this state leads to an increase in price on account of the purchasing penalty, they prefer not to vote. On the other hand, when the valuation is low, customers are

[9]Reverse voting systems were originally developed in social choice theory in the economics literature, with the earliest use in Athenian democracy (Hansen (1999)). In modern times, the EU is considering the use of such systems.

unlikely to buy the product, and voting for the product earns them the immediate reward with no other relevant consequences. Hence, in this system customers can be interpreted as voting *against* the product, since casting a vote signals a low valuation.

Before the voting step, the optimal announcement (r, ρ_{v_o}) maximizes the weighted sum of the profits-to-go, minus the expected cost of rewards $\sum_{\nu_o} Pr\left\{\nu_o|\bar{x}_R^*\right\}\cdot\left(\rho P_R^{\nu_o *}-c\right)\cdot\bar{F}_{\nu_o}\left(\rho P_R^{\nu_o *}\right)-rF\left(\bar{x}_R^*\right)$. As before, we study the interesting case when developing the product is profitable.[10] Equilibrium strategies are provided in Appendix A (Page 110).

Theorem 4. *In a voting system with reverse voting there always exists an informative equilibrium, i.e. where* $F\left(\bar{x}_R^*\right)\in(0,1)$.

Systems with reverse voting can acquire information where traditional systems with purchasing discounts could not. To understand this drastic reversal, it is instructive to examine the incentive proposition for voters in both systems. In voting systems with purchasing discount, customers are asked to incur a costly action (voting) in order to signal a *high* valuation, but the firm's ex-post optimal response after such a signal (to charge a higher price) is against the customers' interest. Thus, there are no incentives to incur the cost of sending this signal. On the contrary, with reverse voting, customers are asked to incur a costly action (voting) in order to signal a *low* valuation, and the firm's response after such a signal (to reward customers and choose a lower price) is in the senders' interest, thus incentivizing them to incur the costs necessary to send the signal and share their information. Note that unlike the voting system with bounded pricing, with reverse voting the firm can freely make the price decision. A priori, this system has the best features of all systems described so far, in that it has full price flexibility and information sharing.

It should be noted that for reverse voting to be effective, voters need to be identified at time of purchase, so that the purchasing penalty can be applied. Arguably, some customers could try to game the system by creating multiple accounts, voting with one account (thus earning the reward) while purchasing the voted product with another account (thus avoiding the penalty). In principle, this type of behavior constitutes a potential problem for a correct execution of reverse voting. In practice, however, firms can drastically reduce such behavior by employing recent web technologies, such as super-persistent cookies, and by also requiring identifying information to cast a vote, such as a combination of email, credit card number, and billing and shipping addresses. Note also that for

[10] Formally, we consider $c_F<\bar{F}\left(\bar{x}_R^*\right)\left(P_R^{0*}-c\right)+\left[F\left(\bar{x}_R^*\right)-F\left(P_R^{1*}\right)\right]\left(P_R^{1*}-c\right)-rF\left(\bar{x}_R^*\right)$.

the purchaser penalty to be an effective incentive, voters just need to be subject to a price increase in expectation, and even a small chance to be identified is enough to deter unwanted behavior, due to the small reward from voting. These considerations suggest that, from a practical standpoint, multiple accounts constitute a very minor issue for an effective implementation of systems with reverse voting.

4.3. **Bounded Pricing or Reverse Voting?** In order to compare the two newly developed voting systems to advise on pricing, it is instructive to reformulate the optimal firm profits in each system into a common profit form. This assumes a particularly interesting structure when the cost of voting, c_v is relatively small compared to the product valuation, which we assume hereafter.[11] This common profit function has two components: the first is the informed profit, PI, which can be interpreted as the profit that a firm earns if it acquires information. The second is the cost of information, CI, which is the cost of incentivizing strategically acting customers to share information.

Formally, the profit of voting system j, where $j \in \{B, R\}$, B denotes the system with bounded pricing, and R denotes the system with reverse voting, can be decomposed as $\Pi_j^* = PI_j - CI_j$ where

$$CI_B = c_v \bar{F}\left(\bar{x}_B^*\right), \qquad CI_R = c_v F\left(\bar{x}_R^*\right), \tag{4.1}$$

$$\forall j \qquad PI_j\left(\bar{x}_j^*\right) = \left(\bar{x}_j^* - c\right) \cdot \bar{F}\left(\bar{x}_j^*\right) + \left(P_l^*\left(\bar{x}_j^*\right) - c\right) \cdot \left(F\left(\bar{x}_j^*\right) - F\left(P_l^*\left(\bar{x}_j^*\right)\right)\right) - c_F, \tag{4.2}$$

and $P_l^*\left(\bar{x}_j\right) = \arg\max_P \left[F\left(\bar{x}_j\right) - F(P)\right](P-c)^+$ (see Appendix A, page 111 for details on obtaining the above common reformulation). We next examine how these two components of profits differ in the two systems.

Cost of Information (CI). The cost of information (Eq. 4.1) is incurred by the firm on different parts of the valuation distribution for the two systems– when valuation is more than the voting threshold in bounded pricing, and when valuation is less than the voting threshold in reverse voting. In a system with bounded pricing, the cost of information is the potential margin that the firm loses in the *high*-valuation contingency because of the firm's commitment to a maximum price. In a system with reverse voting, on the other hand, CI is the expected value of rewards to voters in the *low*-valuation contingency. In both cases, the cost of information increases in the cost of voting.

[11]Formally, $c_v \leq \arg\max_P \left[F\left(\bar{x}_j^*\right) - F(P)\right](P-c)^+$, $j \in \{B, R\}$.

When low valuations are more likely, reverse voting systems end up paying out too many rewards, whereas when high valuations are more likely, bounded pricing systems have their margins crippled by the maximum price commitment. The next theorem formalizes this effect.

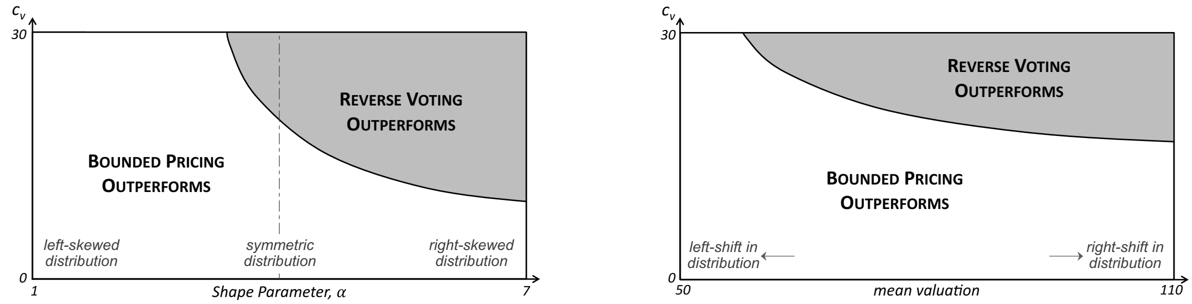
Theorem 5. *Take a valuation distribution f, and let $\bar{x}_B^f$ and $\bar{x}_R^f$ be customer equilibrium voting strategies; then for every valuation distribution g such that $g \underset{fosd}{\succ} f$, the cost of information first-order increases for bounded pricing systems and first-order decreases for reverse voting systems, $c_v F\left(\bar{x}_R^f\right) > c_v G\left(\bar{x}_R^f\right)$ and $c_v \bar{F}\left(\bar{x}_B^f\right) < c_v \bar{G}\left(\bar{x}_B^f\right)$. Further, if f has a non-decreasing hazard rate, then a shift in the valuation distribution, $h\left(x\right) = f\left(x-k\right)$, $k>0$, increases the cost of information for bounded pricing systems, $c_v \bar{F}\left(\bar{x}_B^f\right) < c_v \bar{H}\left(\bar{x}_B^h\right)$.*

Roughly speaking, Theorem 5 shows that moving probability mass from lower to higher valuations (from f to g) favors reverse voting systems. Managerially, this suggests that reverse voting systems work best when the valuation distribution *has a higher mean and/or is right-skewed*. Next, we consider the second component of profits, the informed profit.

Informed Profit (PI). Interestingly, the informed profit (Eq. 4.2) is the same function of the voting threshold, $\bar{x}_j^*$, in the two systems. This voting threshold determines the information that the firm can obtain from voting, and consequently also determines the informed profit. In particular, some voting thresholds get more useful information than others (as an extreme case, thresholds at the boundaries of the distribution's support bring no information or increase in profits). Now, in bounded pricing systems the firm can freely choose this threshold (by appropriately setting the discount and price bound, δ_B and $\bar{P}_B$, see Appendix A, page 111), whereas in reverse voting systems the firm has no control of the same, which is determined by the payoff-dominant equilibrium. This control over the customer voting threshold and consequently of the quality of information gives an advantage to the bounded pricing system. In particular, it can always, at the very least, acquire the same information as with reverse voting, with the potential of doing better. Formally, the thresholds are:

$$\bar{x}_B^* = \arg\max_{\bar{x}_B} \left(\Pi_B\left(\bar{x}_B\right)\right), \tag{4.3}$$

$$\bar{x}_R^* = \arg\max_{\bar{x}_R} \left[\bar{F}(\bar{x}_R)\left(x - P_h^*(\bar{x}_R)\right) + \left[F(\bar{x}_R) - F(P_l^*(\bar{x}_R))\right]\left(x - P_l^*(\bar{x}_R)\right) + c_v F(\bar{x}_R)\right], \tag{4.4}$$



(a) Changing Valuation Skewness
A=30, B=130, $\alpha+\beta$=8, c=0

(b) Shifting Support
B-A=100, α=4, β=4, c=0

FIGURE 4.3: Comparison of Voting Systems to advise on Pricing Decisions

where $P_h^*(\bar{x}_R) = \arg\max_P \, min\left(\frac{\bar{F}(P)}{\bar{F}(\bar{x}_R)}, 1\right)(P-c)$, and the next theorem captures the informational advantage of bounded pricing.

Theorem 6. *There always exist a purchasing discount and a maximum price in the bounded pricing system such that its informed profit is at least as high as that of the system with reverse voting. Further,* $\lim_{c_v \to 0} PI_B - PI_R \geq 0$ *and* $\lim_{c_v \to 0} \Pi_B^* - \Pi_R^* \geq 0$.

The above result shows that the system of incentives set up with bounded pricing not only helps the firm acquire information, but it also allows the firm to acquire the right kind of information. If the cost of voting is low, thereby muting the cost of information component of profit, the higher informed profit in bounded pricing makes it a better system.

Taken together, our results show that the informed profit component always favors bounded pricing, whereas the cost of information component favors bounded pricing or reverse voting depending on the skewness of the distribution. Managerially, we expect that *bounded pricing is preferred when the cost of voting is low, whereas* reverse voting *is preferred when the mean valuation is high and/or the distribution is right skewed.*

Figure 4.3 illustrates these effects.[12] Panel (a) shows that a reverse voting system is preferred with right-skewed valuation distributions, in particular when the cost of voting is substantial. Panel (b) illustrates the consequences of different mean valuations, obtained by shifting the distribution. Interestingly, a right-shift in distribution can also be interpreted as a decrease in unit cost, c. Hence, with increasing mean and/or lower unit costs, reverse voting systems are preferred.

[12]In these examples, customer valuation is beta-distributed, allowing us to change skewness and support by adjusting the shape parameters α, β, A and B. We change the skewness of the distribution while keeping $\alpha+\beta$ constant, which can be thought of as looking at the possible range of priors that a firm engaging in pre-game market research could start with, keeping the amount of market investment constant.

5. Numerical Study

In this section we quantify the benefits of voting systems for two representative products from firms employing voting systems. We consider designs for two new products– a high-value product, the "Chesterfield Leather Sofa", and a low-value product, the "Glass Table Lamp" (Figure A.1 in Appendix A). Table 5.1 reports the parameters associated with each product and describes the methodologies and sources employed to estimate them. Since no data is available about the valuation distribution for these products, we investigate the profitability of voting systems by employing a range of plausible values for the mean, skewness, and variance of the valuation distribution. We begin by presenting the results for voting systems to advise on development decisions, and next for voting systems to advise on pricing decisions.

5.1. **Profitability Gain for Voting Systems to Advise on Development decisions.** Figure 5.1 shows the gain in product-level gross profits when voting is used to advise development. Panels (1) and (2) consider different levels of mean valuation and Panels (3) and (4) consider the costs of voting.

Panels (1) and (2) show that the profit gain from deploying voting systems is substantial, but varies dramatically depending on mean customer valuation. When mean valuation is a smaller multiple of the production costs, or the profit potential is smaller, the gains are higher– reaching as high as 50%. The effect is more pronounced for the low value product. Skewness of the valuation distribution has a substantial impact on profit gains, especially for low-value products with low valuation. Taken together, the gains from deploying voting systems to advise development are most pronounced when the profit potential of the product is smaller, that is, when the costs (production, shipping and development) are comparable to the customer valuation and the valuation distribution is left skewed.

Panels (3) and (4) show that a change in the cost of voting c_v has almost no impact on profit gains for the high-value product, but it has a substantial and roughly linear impact for the low-value product, with a \$1 increase in c_v resulting in as much as a 6% reduction in profit gains. Higher uncertainty in the valuation distribution increases the benefits from deploying voting systems. For the low-value product, a 20% increase (decrease) in variance of the valuation results in a 5-6% increase (decrease) in profit gains, compared to less than 1% for the high-value product.

Parameter		***Lamp***	***Sofa***	***Estimation Method/Sources***
Unit Cost, c	production cost, c_p	\$39	\$600	Based on quotes obtained from alibaba.com
	shipping cost, c_s	\$11	\$141	Sum of Port-to-Port rates (calculated for the Los Angeles-Wenzhou route (www.freight-calculator.com) using quantity per-shipment estimates) and last-mile rates (B2B pricing estimates from DHL). All values verified against freight rates at retailer MyFab.
Development Cost, c_F		\$7.3	\$36.6	Based on minimum order quantity quotes from alibaba.com. Pro-rated over the expected number of units sold (sales data from MyFab).
Voting Cost, c_v		\$3.52-6.08		Range of frictional cost estimated in Hann and Terwiesch (2003)

TABLE 5.1: Parameter values and Sources

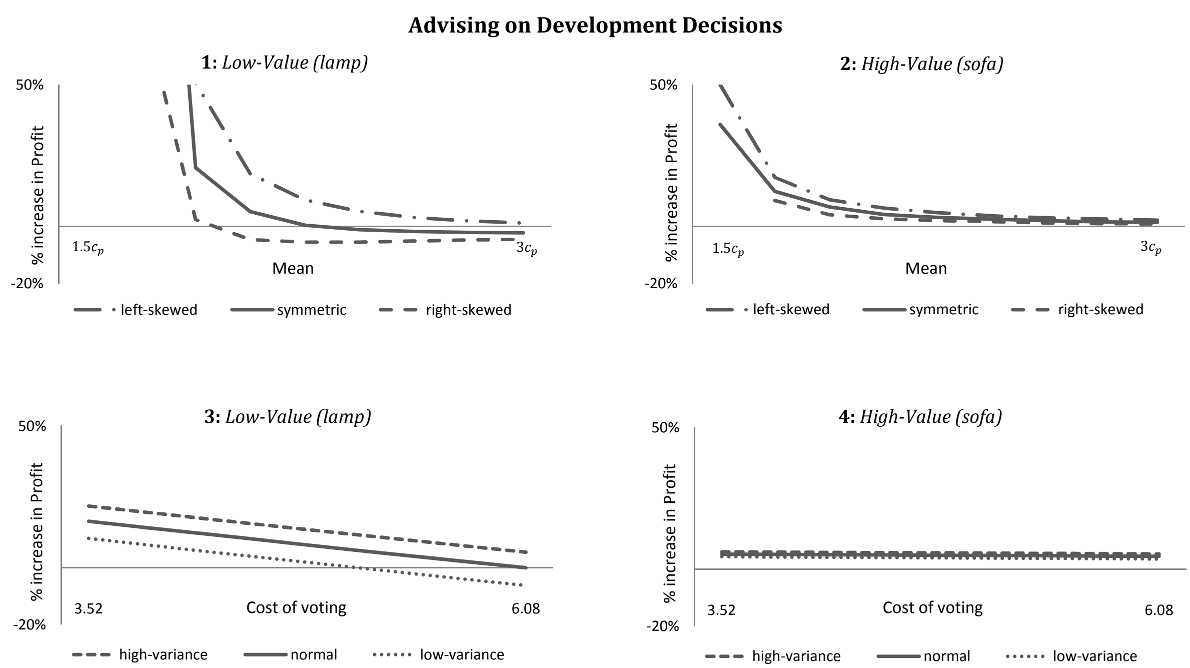


FIGURE 5.1: Profitability Gains from Voting Systems to Advise on Development
Panels 1 and 2: Cost of voting c_v= \$4.8 (average estimate from Hann and Terwiesch (2003)). Valuation, $X \sim$ Beta with shape parameters $(\alpha, \beta) = (4, 4)$ in the symmetric case and $(3, 5)$ or $(5, 3)$ for skewed cases.
Panels 3 and 4: Mean Valuation $\mathbb{E}[X]$= 2*Production cost. Valuation is symmetric, $X \sim$Beta with shape parameters $(\alpha, \beta) = (4, 4)$ as the base case, High/Low variance is $\pm 20\%$ variance.
In all panels: Lower bound of valuation distribution, A, is the production cost of the product in question.

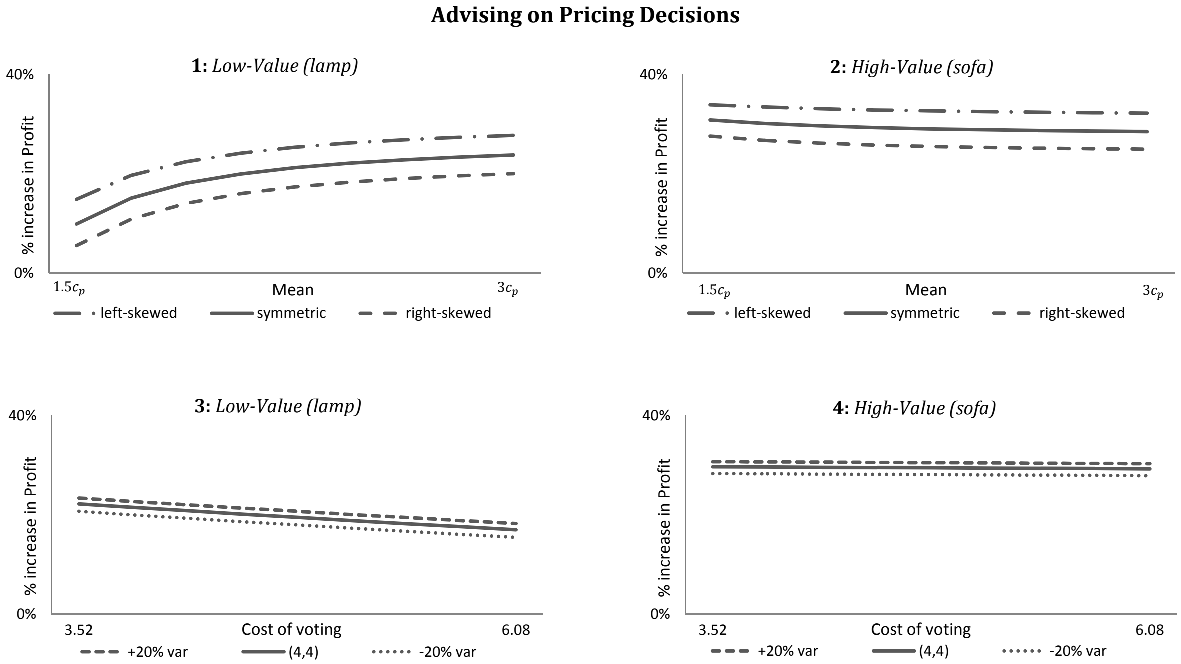


Figure 5.2: Profitability Gains from best Voting System to Advise on Pricing
Panels 1 and 2: Cost of voting c_v= \$4.8 (average estimate from Hann and Terwiesch (2003)). Valuation, $X \sim$ Beta with shape parameters $(\alpha, \beta) = (4, 4)$ in the symmetric case and $(3, 5)$ or $(5, 3)$ for skewed cases.
Panels 3 and 4: Mean Valuation $\mathbb{E}[X]$= 2*Production cost. Valuation is symmetric, $X \sim$ Beta with shape parameters $(\alpha, \beta) = (4, 4)$ as the base case, High/Low variance is $\pm$20% variance.
In all panels: Lower bound of valuation distribution, A, is the production cost of the product in question.

Overall, these numbers show that voting systems can increase product profits substantially, with the maximum benefits arising for products that have costs comparable to customer valuations.

5.2. **Profitability Gain for Voting Systems to Advise on Pricing decisions.** Figure 5.2 illustrates the gains obtainable in product-level gross profits from systems to advise on pricing decisions, the panels being organized as before.

Panels (1) and (2) show that in contrast to their use for advising development, voting systems to advise pricing are most beneficial for products when mean valuation is substantially higher than the costs involved. For low-value products, gains increase from about 10% to 25% as mean valuation increases in comparison to costs. For high value products, the gains are stable at around 30%. Skewness in the valuation distribution has some impact on the profit gains, with a mildly left-skewed (right-skewed) distribution resulting in a 3-5% increase (decrease).

Panels (3) and (4) show that a change in the cost of voting affects only low-value products, its effect being roughly linear in c_v, with a \$1 increase in c_v resulting in a $\sim$2% loss. This reduction is lower compared to voting systems to advise on development because here the firm has the ability to lower the price and sell to customers even in the low contingency, where the other voting systems would halt development and make no profit. Higher uncertainty has a small positive impact on profit gains, with a 20% increase (decrease) in variance resulting in a 1-1.5% increase (decrease).

These results show that voting systems employed to advise pricing can increase firm profits substantially: a firm using them can expect profit gains as high as 25-30% with the maximum gains when products have high profit potential, that is, when customers value products substantially higher than costs.

6. Discussion

Voting systems are a novel and innovative way of engaging customers in firm operations while acquiring important customer preference information. While these customer engagement and information acquisition aspects make them prime candidates for many online retailers, our analysis shows that their successful deployment and effective use requires an understanding of the incentives of all players involved. In particular, our analysis demonstrates that the intended use of acquired information completely changes the appropriate system design– while the design of offering incentives to *vote for products* is an effective information acquisition mechanism for incentive-aligned decisions, new system designs are required when the interests of the parties are misaligned, as in the case of pricing decisions. In particular, systems with reduced flexibility or systems that engender *voting against products* are likely to be most effective in allowing the firm to best match its operational decisions to the demands of its customers.

In our model we study the use of customers voting systems to advise development or pricing decisions on a single product. Our model fully extends to situations with multiple products as long as the decisions to be made on different products are largely independent from one another. This is true, for example, when products of different categories are put up for voting–as it often happens in the furniture industry–and also when customers consider buying multiple products–as it is likely the case for the cheap t-shirts sold at Threadless. On development decisions, for example, the focus is usually not on selecting the most promising item(s), rather on selecting all items that are promising enough. Future work could expand our analysis to consider the case of multiple substitute products.

While we present the analysis in the paper in the context of voting systems in online retail settings, our model of customer voting systems can also be easily adapted to rapidly growing non-profit communal venture funding. Collectively referred to as crowdfunding models, the most prominent of which is Kickstarter (cf. Pogue (2012)), these models have attracted widespread attention, including legislative encouragement in the JOBS act of 2012 (HR 3606), and have become a key part of every entrepreneur's toolkit. Like voting systems, in crowdfunding, customers have a costly action to signal their interest in a product–they are asked to pledge money for new products under development. As in voting systems, the decision of the firm to invest in developing the product is based on the level of support received by customers. Finally, as in voting systems, customers that support new products are compensated with a reward, often in form of a discount (or additional feature/customization) upon purchase of the product. This is equivalent to setting $\delta_D = 1 - \frac{d-s}{P_D}$ in our model of Section 3, where s is the sum pledged by the customer and d is the discount (or the value of the additional feature) promised by the firm.

Insights from our model can therefore also be applied to crowdfunding. Specifically, our analysis suggests that crowdfunding systems are likely to be informative only when customers pledging are granted some rewards. The benefits of voting systems are likely to be most salient for products that have high development and unit costs. Further, they might not be most effective in systems with high valuation uncertainty. Most importantly, while pledges and purchaser benefits are effective in advising product development, we caution against the use of these systems to support product pricing and venture profitability estimations.

While our analysis above uses development as a canonical decision where firm and customer incentives are aligned, we believe our insights extend to other decisions with similar incentive structures. For example, building capacity, stocking inventory, and building logistics or after-sales support capabilities are all decisions where in the case of high demand, both the firm and customer desire the same actions. On the other hand, decisions on promotions, bundling, discounting, and retail execution are all akin to pricing, where in high-valuation states of the world the firm and customers desire different actions. The first analysis of voting systems provided in this study offers key guidelines on system use; further study, in particular on the behavioral aspects of voters' engagement, holds the promise of helping firms fully realize the potential of these innovative business models.

Part 2. Operational Advantages and Optimal Design of Threshold Discounting Offers

We study the use of threshold discounting, the practice of offering a discounted price service only if at least a given number of customers show interest in it. In recent years, firms like Groupon and others in the newly created multi-billion-dollar online deals industry have popularized this approach. We model a capacity-constrained firm servicing a random-sized population of strategic customers in two representative time periods, a desirable hot period and a less desirable slow period. A comparison with the more traditional approaches typically employed in these circumstances (slow period discounting and closure) reveals that threshold discounting boosts the firm's operational performance on account of two advantages. First, the contingent discount incentivizes slow period consumption when the market for the service is large and reduces supply of the service when the market is small, in effect allowing the firm to respond to the service's unobserved market potential. Second, activation of the threshold discount signals the market state to strategic customers, supplying them with additional information on service availability, and inducing them into self-selecting the consumption period to one that improves the firm's capacity utilization and profit. Unlike in a typical setting with strategic customers, strategic behavior in our setting helps the firm, and a higher fraction of strategic customers in the population increases the firm's profits. We consider alternate deal designs, and we find that the best designs compromise the service provider's flexibility in order to provide strategic customers with clear offer terms. We conclude with a numerical study calibrated on data from the opera house Teatro Regio in Torino, Italy, where we consider a number of market and customer behavior scenarios to estimate that threshold discounting improves firm profits over traditional approaches by as much as 29% (6.1% on average) and makes the firm's profits more resilient to increasing levels of market uncertainty.

7. Introduction

Firms often operate in environments in which they must serve highly variable demand with capacity that is fixed in the short term. This setting has particularly acute consequences for service firms, as spare capacity in low-demand periods typically cannot be used to serve customers in high-demand periods. Prominent industries that struggle with this problem include the movie theater industry ($10.4 Billion of revenues in 2014), the restaurant industry ($709 Billion of revenues in 2014),[13] and a wide variety of retail services, such as beauty salons, bowling clubs, museums, opera houses, etc.

Over the last decade, the rise of online customer engagement technologies has provided service providers in these industries with new tools to interact with customers, most notably online discounted deals. The most famous online deal website is certainly Groupon: founded in late 2008 and the first of an innovative breed of firms, it grew by 2,241% in its second year of operation–faster than celebrated firms like Amazon or Ebay–and went public in 2011, raising $700 Million to become the largest IPO by a US Internet company after Google.[14] Groupon's growth was fueled by the use of an innovative discount structure, in which customers could purchase retail services with a substantial discount, but the discount was valid only if a certain number of customers showed interest in the offering. From here on, we refer to deals where discounts are contingent on a threshold number of customers as *threshold discounting* offers.

The popularity of threshold discounting offers has increased in the last few years together with the industry around them, becoming an almost essential feature for the hundreds of websites that have spawned all over the world trying to imitate Groupon's business model.[15] Despite the enormous popularity of threshold discounting, there is little agreement on the benefits associated with such offers. Some commentators argue that their advantage lies in "...driving bargains using economies of scale in local markets...", others think that the benefit is "...a combination of network effects and economies of scale...", and someone goes as far as to say that ..."there is no (...) substance to the idea that a threshold of buyers has to be reached before a deal is 'activated' (...) there's no real benefit

[13]Sources: http://goo.gl/EaZt2, http://goo.gl/qZUKX. Data for the United States.

[14]See "Groupon's IPO biggest by U.S. Web company since Google", Reuters, November 4, 2011, http://goo.gl/h8VFj, and "Groupon IPO: Growth Rate Is 2,241%", The Wall Street Journal, June 2, 2011, http://goo.gl/UFFwK.

[15]See "Group Buying vs. Social Buying", Ingenesist, May 24, 2010, http://goo.gl/ewSb, "Getting In With Groupon and LivingSocial: What You Need to Know About These Fast-Growing Sales Drivers", National Federation of Independent Business, September 2012, http://goo.gl/DpqKl, "Retail Top 100 2012, No. 33: Groupon/LivingSocial/etc.", Retail Customer Experience, December 2012, http://goo.gl/nGm3b, and http://startagroupbuy.com/.

in that...".[16] The few academic articles on the topic have taken a marketing perspective and praised threshold discounting offers for their ability to improve sales and profit (see Section 8). Recently, the debate on the benefit of threshold discounting schemes has been muddied as Groupon, the pioneer in their use, has stopped using these offers, implicitly reinforcing the position of those who believe that they deliver no real value. Given that the source of the benefits of threshold discounting is not well understood, it comes as no surprise that there is even more confusion when it comes to designing these deals–in fact, to the best of our knowledge, no analysis has been undertaken in the academic literature on the timing of events in threshold discounting offers or on their impact on firm profit.

The objective of this paper is to understand the operational advantages of threshold discounting offers and to provide recommendations on their optimal design. We consider a typical situation in which threshold discounting is used, i.e., a capacity-constrained firm offers his services to a random-sized population of strategically-acting customers who prefer to be served on a desirable "hot" time period over a less desirable "slow" time period (e.g., a theater on Saturday versus Monday evening), with the degree of preference for the hot period over the slow period varying across the population. Demand is thus variable but substitutable between two vertically differentiated services, the hot-period service and the slow-period service.

A traditional approach used by firms in this context is to either close on the slow period, or price discriminate–i.e., open on the slow period at a discounted price. A comparison with threshold discounting reveals that threshold discounting outperforms the traditional approaches on account of two effects. First, by setting an activation threshold, threshold discounting endows the firm with a built-in, demand-responsive mechanism (which we refer to as *responsive duality*) that matches different market states with appropriate pricing/closing decisions, resulting in higher capacity utilization and better fixed costs management. Second, threshold discounting induces a *strategic scarcity effect* that increases customers' responsiveness to slow-period discounts by exploiting their strategic behavior: inducing them into self-selecting their consumption period to one that better serves the firm's interests of managing capacity and margins. Notably, the superior performance of threshold discounting over the traditional approach holds even in the absence of economies of scale or network consumption effects, the only benefits of threshold discounting that have been studied.

[16] See "Groupon: What a Deal", July 2011, http://goo.gl/I4kNp; Forbes, October 2011, http://goo.gl/doIJ2; Groupbuyingsites.co.uk, December 2011, http://goo.gl/YfjHF.

The impact of strategic customers on the profits of a firm employing threshold discounting is particularly interesting. The literature on strategic customers has largely found that they reduce a firm's profit because they time their purchases in order to get lower prices, thus reducing margins for the firm. In contrast with the literature, we find that in our context having more strategic customers is *beneficial* for the firm, that is, the higher the fraction of strategic customers in the population, the higher the firm's profit.

We next study various dimensions of the design of threshold discounting schemes. We first consider if the firm should commit upfront to the number of customers required for the deal to be active, or instead decide on the deal activation only after observing the subscription level. Interestingly, we find that postponing the deal-activation decision to incorporate the market information contained in the subscription level is harmful to the firm. We then investigate the best time for the firm to reveal whether the threshold has been reached–specifically, if this should be before both periods begin or not–and find that early disclosure is the superior design. We also compare time-restricted threshold discounts with oft used unrestricted discounts, and find that time-restricted discounts are superior. Finally, we show that under certain conditions the preferred *committed threshold, early disclosure*, *time-restricted* design of threshold discounts can be further improved by offering targeted discounts that reduce lost margins while retaining all the operational advantages of these deals.

We conclude with a numerical study calibrated on real data from the opera house Teatro Regio in Torino, Italy. We consider different market size distributions, customer preference distributions, and cost structures. In the 300 scenarios examined, we find that threshold discounting increases firm profits by as much as 29% (average 6.1%) over the profit earned with the traditional approach. Moreover, threshold discounting is more resilient to increasing market uncertainty than traditional approaches: while higher uncertainty typically lowers firm profits, it reduces profits to a lesser extent or even increases profits for threshold discounting.

Our work makes several contributions. This is the first study to examine the operational advantages of threshold discounting, a popular phenomenon that has spawned a multi-billion-dollar online deals industry. We identify two novel operational advantages of these discounts–the contingent nature of the discount that reduces supply-demand mismatches, and the ability of these discounts to use strategic customer behavior to their advantage. In contrast with the strategic customer behavior literature in operations, we show that in our setting, strategic customer behavior is beneficial. Further, we provide clear prescriptions about the design and use of threshold discounts–specifically, that a service provider is always better off offering a *committed threshold, early disclosure*, *time-restricted* threshold discount as opposed to the variety of other designs touted by deal intermediaries and designers, and we show that focused threshold discounts can improve profits even further. Our analysis cautions potential users of these discounts to the incentive conflicts inherent in the current modes of offering these discounts through intermediaries. We conclude our study with a quantification of the profit increase from offering threshold discounts (up to 29%) and we find that these benefits are highest in situations characterized by high market uncertainty. Overall, our paper argues that threshold discounting is an overlooked method for managing capacity that, if used correctly, can substantially improve a firm's performance.

8. Literature Review

Our work is related to three streams of literature: group-buying and quantity discounts, strategic consumers, and demand manipulation via pricing.

Group-buying and quantity discounts - In the early 2000's, several group buying websites like Mercata.com, LetsBuyIt.com, and Mobshop.com were founded with the objective of aggregating the buying power of customers to obtain quantity discounts. Anand and Aron (2003) model these group buying practices with a firm offering a price-quantity menu to customers, and find that group buying is better than a simple fixed price only when either demand uncertainty satisfies certain conditions, or economies of scale are coupled with production postponement. In a similar spirit, Chen et al. (2007) study group buying auctions, where a firm commits to a price-quantity function and customers arrive stochastically and bid their reservation price; they find that group buying is better than simple fixed pricing only in the presence of economies of scale, or when the firm is risk-seeking. Unlike our paper, the above works do not consider vertically differentiated services or threshold discounting schemes.

More recently, after the surge in popularity of online threshold discounting deals in the service retail industry, some authors have devoted attention to study this novel phenomenon. Edelman et al. (2011), using a formal model, study the business proposition of Groupon (while ignoring threshold discounting offers). They highlight two advantages of Groupon-like offers: the ability to use discounted price to attract low valuation customers, and the ability to increase customers' awareness of the business–in the same vein as an advertising campaign. The use of threshold discounting is briefly discussed in Arabshahi (2010), which argues that the existence of an activation threshold aims to reassure customers about the quality of the service they're buying, possibly because the discount is granted due to economies of scale and not poor quality. In their study of the soft tools used by Groupon to improve their business, Byers et al. (2011) suggest that the activation threshold could be thought of as inducing a word-of-mouth effect among customers, which is formally studied in Jing and Xie (2011), who argue that informed players act as sales representatives with their friends in an attempt to reach the threshold. More recently, Chen and Zhang (2014) study threshold discounts and show that, under some conditions, these offers are the optimal mechanism to price discriminate a population containing two types of customers, one of which is of random size. While similar to ours, none of these papers take an *operational* perspective on threshold discounting schemes, consider inter-temporal demand substitution, or provide explanations for why major players have discontinued threshold discounting offers. While similar to ours, none of these papers take an operational perspective on threshold discounting schemes, consider inter-temporal demand substitution, or attempt to address the issue of how to best design threshold discounting offers.

Strategic customers - A few decades ago, Coase (1972) conjectured that a monopolist selling a durable good would eventually lower its price down to marginal cost when facing infinitely patient consumers. Recent years have seen renewed interest in the operational implications of customer strategic behavior. Most of the work has focused on strategic purchasing delay on the part of customers when a firm sells a finite inventory of a durable good and may change the price over time. Su (2007) considers customers with different valuations for the product and degrees of patience, and develops insights on how the interplay of these characteristics affects the firm's pricing policy and profit. Liu and van Ryzin (2008) study how the capacity choice of a firm can be used to induce a rationing risk on risk-averse strategic customers and limit their strategic purchasing delay. Cachon and Swinney (2009) consider a setting in which the firm cannot commit in advance to prices, and they study the value of quick response strategies to mitigate the negative effect of strategic purchase delays on the part of customers when there are different classes of customers, while Cachon and Swinney (2011) explore the interplay of quick response and enhanced design in fast fashion systems. Aviv and Pazgal (2008) consider both pre-announced and contingent pricing strategies, and they provide recommendations for when these different approaches should be used if both are viable. Su and Zhang (2008) study the value of both quantity and price commitment, and show how a decentralized supply chain can exploit the inefficiencies of decentralization as a commitment device to indirectly implement price and quantity commitment strategies, even when commitments are not credible. Strategic customers have also been studied in other situations, including consumer stockpiling in Su (2010), opaque selling strategies in Jerath et al. (2010), conspicuous consumption in Tereyağoğlu and Veeraraghavan (2012), product variety in Parlaktürk (2012), online click-tracking in Huang and Van Mieghem (2012), pre-orders in Li and Zhang (2013), social comparisons in Roels and Su (2013), early buyer reviews in Papanastasiou et al. (2013), and the informative power of queue length in Veeraraghavan and Debo (2009, 2011) (see Netessine and Tang (2009) for more references).

Like many of the above papers, our customers time their purchases accounting for the strategic behavior of other players. Unlike the above papers, however, we explore the consequences of such strategic behavior in a novel setting, a firm that employs threshold discounting offers in the context of vertically differentiated service periods. The implications of strategic behavior in our context are unexpected and in contrast with the main findings from this large literature.

Demand manipulation via pricing - This body of literature deals with situations in which a capacity-constrained (or inventory-constrained) firm can use the pricing decision to reduce the supply-demand mismatch. The most relevant for us is the stream that deals with peak load pricing, i.e., the pricing of economically non-storable commodities whose demand varies periodically. Many authors have analyzed the problem under different circumstances, including deterministic demand (Steiner (1957)), rationing rules (Visscher (1973)), the use of different technologies (Crew and Kleindorfer (1976)), and uncertainty on both the demand and the supply side (Kleindorfer and Fernando (1993)). See Crew et al. (1995) for a survey of the literature on peak load pricing. To the best of our knowledge, only one work has considered inter-temporally substitutable consumption (Crew and Kleindorfer (1986)), but assuming a deterministic demand, while demand uncertainty plays a key role in our setting.

Other works have considered the role of pricing as a way to improve operational performance. All the literature on revenue management, for instance, focuses on this topic (see Talluri and Van Ryzin (2005) for a survey). In his paper on price dispersion, Dana Jr (1999) shows the operational benefit of shifting demand across time periods by rationing the number of seats offered at a lower price, even when firms cannot predict the peak time. In other settings, Lus and Muriel (2009) find that pricing is more effective than technology choices at balancing supply and demand when a firm sells highly substitutable products, and Boyacı and Özer (2010) show how advanced selling and pricing can be jointly used to reduce the demand-supply mismatch.

Our paper departs from the existing literature in that we study a way to reduce the supply-demand mismatch through a novel pricing approach: namely, we study the use of correctly designed threshold discounting offers in the presence of strategic customers.

9. The Model

9.1. **Preliminaries.** Consider a capacity-constrained service provider that offers his services in two time periods to a random-sized population of strategically acting customers. Customers prefer to be served in a desirable "hot" time period over a less desirable "slow" time period (e.g., a theater on Saturday versus Monday evening) and have varying degrees of preference. After briefly describing the model, we first examine the traditional approaches typically employed by firms in similar circumstances–a choice between closing down or discounting on the slow period–and then we compare the result with threshold discounting offers as popularized by online deal sites such as Groupon, LetsGroop, BigDeal, Ihergo, etc.

Service Economics. We model provision of the service in two representative time periods: in a hot period preferred by customers, at a price r_h, and in a less preferred slow period.[17] The service provider has capacity to serve at most k customers during each service period, but has the flexibility to shut down or choose any price in the slow period. When offering the service in a given period, the service provider incurs fixed costs c_F (for employees, utilities, etc...) plus an additional expense of c for every customer served. The costs are not too prohibitive to preclude profits, $c_F < k(r_h - c)$. When demand outstrips capacity, the provider rations capacity randomly amongst customers.

Customers. The service is made available to a market comprised of atomistic customers of aggregated size $\tilde{x}$, where $\tilde{x}$ is an unobserved random variable with support $\mathbb{R}_+$, cumulative distribution function G, and survival function $\bar{G} = 1 - G$. Customers value the service in the hot period at v_h, $v_h > r_h$, higher than their value in the slow period, $\tilde{v}_s$, which varies across customers–i.e. customers differ in the degree to which they prefer the hot period over the slow period. Each customer's slow period valuation, $\tilde{v}_s$, is privately known only by the customer herself; it is drawn from a continuous distribution, with cumulative density H, survival function $\bar{H}$, and support $[\underline{v}, v_h)$. Customers desire to consume the service in at most one period, and they can choose their time of consumption strategically–i.e. each customer takes into account the choices of other customers, thus forming expectations of the service availability in different periods. Customers use these refined beliefs, in addition to the provider's announced shutdown and pricing decisions, and the private information on the slow period valuation to make their consumption timing decisions.

[17] We place no restrictions on which period comes first.

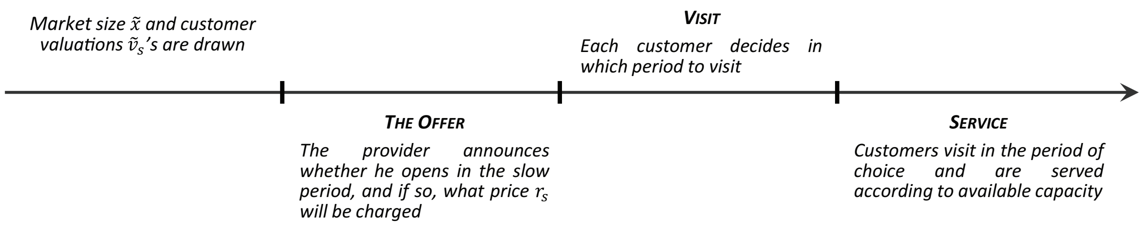


FIGURE 9.1: Timeline for the Traditional Approaches

The setup described above corresponds to a wide variety of consumer services such as movie theaters, spas, opera houses, etc. Each of these services share the key characteristics of our setup–desirable and less desirable service periods, single consumption, and per-period capacity that is fixed in the short run.

9.2. **The Traditional Approach: Seasonal Closure or Price Discrimination.** Traditionally, service providers either shut down in slow periods or remain open but offer a discounted price. For example, in many cities of mainland Europe where fixed costs of operation are high, restaurants and museums are typically closed on Monday.[18] On the other hand, in London, service providers often stay open on Mondays, but offer discounts and promotions to attract customers.[19] Formally, the service provider first decides whether to offer the service in the slow period and, provided the service is offered, what price to charge. The sequence of events is provided in Figure 9.1, and the equilibrium solution is provided in the Appendix (Section B.1).

The service provider's decision whether to open in the slow period is driven by a comparison of the profits from closing on the slow day (seasonal closure) and the profits from opening and offering a discount (price discrimination). Closing the business in the slow period implies that the service provider gives up some of his capacity–capacity available during the slow period–in order to save on the fixed cost c_F. In this case, customers visit during the hot period, and the service provider serves them up to capacity. The expected profit with this approach is

$$\Pi_c = (r_h - c) \int_0^{+\infty} \min(k, x)\, \mathrm{d}G(x) - c_F, \tag{9.1}$$

where the subscript c stands for closure.

[18]See http://goo.gl/M52do.

[19]For example, Stickyfingers Café, Queens bowling, The Lexi Cinema, and Cavendish Conference Venues run "Monday madness" promotions, reducing their prices on Mondays, when they expect fewer customers.

Alternatively, the service provider may offer the service in both hot and slow periods, albeit at a lower price $r_s \leq r_h$ in the slow period–where the subscript p stands for price discrimination. Under this strategy, a customer's consumption timing best-response is driven by a trade-off between the higher utility she derives from the hot period on the one hand, and the better prices in the slow period on the other, both adjusted by her rational expectation regarding service availability in each time period.

Formally, a customer visits during the slow period iff her slow period valuation for the service is higher than a threshold valuation $\hat{v}_p(r_s)$, which is the valuation that makes a customer indifferent between the two service periods, and is given by

$$(9.2) \qquad (v_h - r_h) \int_0^{+\infty} \min\left(1, \frac{k}{H(\hat{v}_p)\, x}\right) \mathrm{d}G_c(x) = (\hat{v}_p - r_s) \int_0^{+\infty} \min\left(1, \frac{k}{\bar{H}(\hat{v}_p)\, x}\right) \mathrm{d}G_c(x),$$

where the LHS (RHS) represents the expected surplus of the customer from visiting during the hot (slow) period, obtained as the product of the service surplus times the expected availability of the service, and where $G_c(x) = \int_0^x u \mathrm{d}G(u) / \int_0^{+\infty} \mathrm{d}G(u)$ is the cdf of the market size from the perspective of an individual customer, i.e. conditional on her existence in the market (see Deneckere and Peck (1995) for the derivation of customer posterior beliefs in these cases). Note that customers' best-response visit strategy $\hat{v}_p(r_s)$ is unique since the LHS and RHS of (9.2) are respectively increasing and decreasing in $\hat{v}_p$ for every price r_s. The expected profit for the provider is given by

$$(9.3) \quad \Pi_p = \max_{r_s} \left[(r_h - c) \int_0^{+\infty} \min(k, H(\hat{v}_p(r_s))\, x)\, \mathrm{d}G(x) + (r_s - c) \int_0^{+\infty} \min\big(k, \bar{H}(\hat{v}_p(r_s))\, x\big)\, \mathrm{d}G(x) - 2c_F \right]$$

$$s.t.\; r_s \leq r_h.$$

The potential advantage of price discrimination is best explained by looking at the effect of a price reduction on profit. To see this, let θ be the discount level set by the firm, such that $r_s = (1-\theta)\, r_h$. The first derivative of the profit under price discrimination with respect to the discount θ can then be written as

$$\frac{d}{d\theta}\Pi_p(\theta) =$$

$$(9.4)\quad \underbrace{-\int_0^{+\infty}\min\Big(k, H(\hat{v}_p(\theta))x\Big)\mathrm{d}G(x)}_{margin\,loss}+\underbrace{\int_0^{+\infty}\frac{d\bar{H}(\hat{v}_p(\theta))}{d\theta}r_h\left(1_{x<k\left(\bar{H}(\hat{v}_p(\theta))\right)^{-1}}-1_{x<k(H(\hat{v}_p(\theta)))^{-1}}(1-\theta)\right)x\mathrm{d}G(x)}_{operational\,effect}$$

where $\hat{v}_p(\theta)$ is short notation for $\hat{v}_p(r_s(\theta))$. A discount has two consequences for the firm: it reduces the margin in the slow period (first component) and it shifts some demand from the hot to the slow period (second component). This second component captures the *operational* effect of a discount: if the discount is not excessive, this demand shift improves capacity utilization by rebalancing demand across the two service periods, thereby reducing the demand-supply mismatch (Section B.2). This rebalancing of demand, often achieved via price reductions in the slower period, is typically referred to in the operations literature as *demand smoothing.* The optimal price is the one that optimally trades off the benefits of higher margins and those derived by demand smoothing. In other words, under price discrimination a firm trades off margins and capacity utilization (see Lemma 7 in Appendix A for more details).

The firm's expected profit with the traditional approach is therefore $\Pi_a = \max(\Pi_c, \Pi_p)$, where Π_c and Π_p are the profits if the firm closes on the slow day or not, defined in (9.1) and (9.3), respectively. While the firm must decide ex-ante whether to close in the slow period (seasonal closure) or open and offer a discount (price discrimination), it is instructive to compare ex-post profits as a function of market realization, to examine when it would have been better to open, and when it would have been better to close. Put differently, the next Lemma provides the strategy that would be followed by an omniscient firm, a firm that could observe the market size from the start.

Lemma 2. *The realized profits under price discrimination are higher than under seasonal closure iff the market size realization is higher than a critical level, x°. This critical market size is greater than capacity k, and it increases in both the fixed costs of opening c_F and the marginal cost c, i.e. $x^\circ > k$ and $\frac{d}{dc_F}x^\circ > 0$, $\frac{d}{dc}x^\circ > 0$.*

Figure 9.2 shows the realized, ex-post profit of price discrimination and closure, together with their difference, as a function of realized market size. When market size is low, closing on the slow day is preferred for two reasons: first, it saves on fixed costs, and second, it prevents the

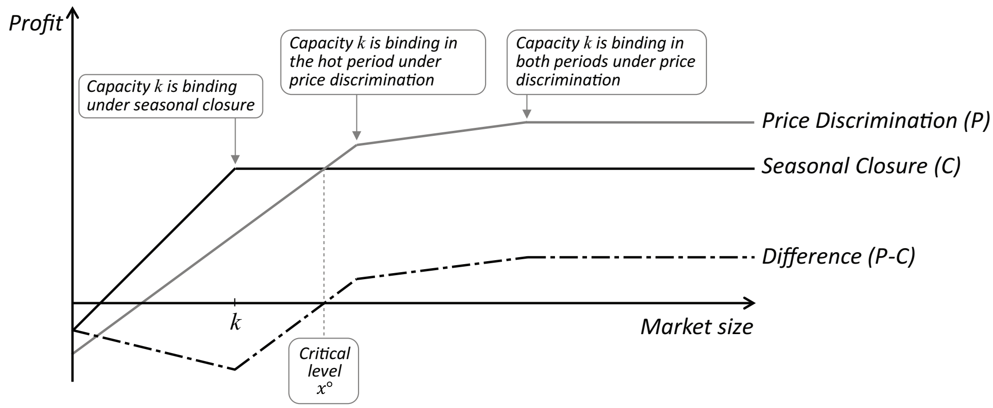


FIGURE 9.2: Profits from Price Discrimination and Closure

lower-priced slow period from cannibalizing sales in the higher-priced hot period.[20] The advantage of closing is the highest when market size is equal to capacity k. Any higher market state results in lost sales with closure, but corresponds to higher sales if the firm is open in the slow period. Eventually, this makes price discrimination preferred to closure when market size is higher than x°. The critical market size x° increases when fixed and variable costs (c_F and c) are higher, because it takes more customers to cover the fixed costs incurred in the slow period, either because the fixed costs themselves are higher, or because slow period margins are lower.

Taken together, the above discussion highlights the key weakness of the traditional approach. The service provider is forced to make an ex-ante trade-off between the preferred strategy for low market states, closing and bearing the risk of losing sales due to limited capacity, and the preferred strategy when the market state is high, opening and bearing the risk of not repaying the augmented fixed costs due to thinner margins. We next examine a threshold discounting scheme, which alleviates this trade-off.

[20]Sales cannibalization is a consequence of the operational effect of discounts (demand shift) that takes place in the absence of capacity shortages; it is, in essence, the harmful side of demand smoothing.

10. Threshold Discounting

Threshold discounting allows customers to visit a firm and avail themselves of the service in the slow period at a discounted price, contingent on enough other customers showing interest in doing the same. In this section, we analyze the benefits of offering such a deal to strategically acting customers.

10.1. **Description and Equilibrium.**

Sequence of Events. Figure 10.1 illustrates the sequence of events for a threshold discounting scheme. At the beginning, the service provider announces a deal: the service will be offered at a discounted price $r_s < r_h$ to all customers who subscribe to the offer, but only if at least n of them end up signing up for it. If less than n customers sign up for the deal, the service provider will close during the slow period.[21] Each customer then decides whether to subscribe to the deal or not.[22] After the subscription deadline passes, the firm communicates whether the deal's activation threshold was reached or not, and therefore whether the deal is active or not. Customers then choose a period to visit, and consume the service.

In order to study this game, it is convenient to break it into two parts: the initial deal offer, and the following continuation game (Fudenberg and Tirole (1991), page 331), in which customers subscribe, the firm reveals the deal outcome, and then customers visit the firm in a period of their choice. This continuation game, which follows the initial deal offer, is strategically played only among customers: in fact, in the deal outcome disclosure stage the firm simply reveals whether the pre-announced threshold was reached (in Section 11.1 we consider an alternate design where the

[21] A weaker form of threshold discounting is one where only the pricing decision is determined by subscribers, and the firm stays open in both periods. In this case the results are very similar, as briefly discussed in Section 13.

[22] We assume that subscriptions are not binding for customers; if subscriptions are binding, i.e., customers are pre-charged the slow period price r_s upon subscription, all our results are identical and in fact the analysis is simpler.

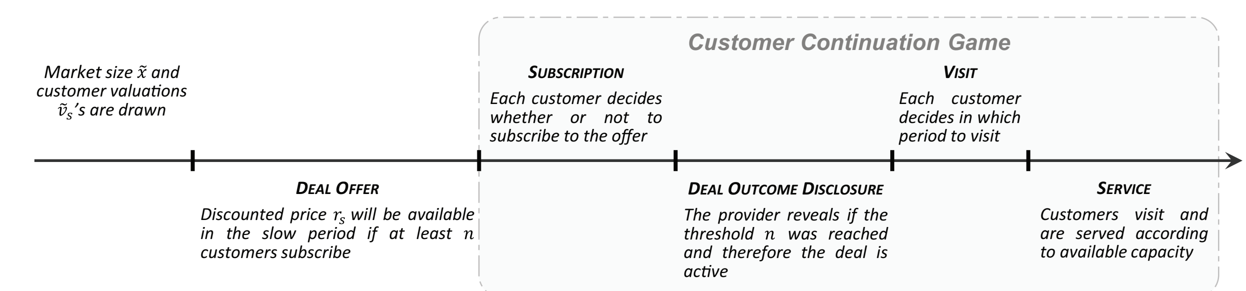


Figure 10.1: Timeline of Threshold Discounting

firm can freely make his activation decision after observing the number of subscribers). In what follows, we first proceed with the analysis of the customer continuation game for a given deal offer (r_s, n), and then we include the deal offer decision made by the firm to find the equilibrium of the full game.

Customer Continuation Game. We examine the best-response strategies of an individual customer starting from the last stage of the continuation game, when she must decide in which period to visit the firm. When the deal is not active, the service is not available during the slow period and therefore she visits in the hot period. When instead the deal is active, she visits in the period in which she expects to obtain the highest surplus. Specifically, the visit strategy ν_i of customer i is a function of her service valuation for the slow period $v_{s,i}$, the price that she will be charged during the slow time period, i.e. r_s if she has previously subscribed to the deal and r_h otherwise, and the expected service availability in each time period. To compute expected service availability, the customer takes into account the vector of visit strategies of all other customers, ν_{-i}, and forms a posterior belief on the realized market state conditional on the information that the deal is active, that is, the posterior distribution of the market size given that at least n customers subscribed employing the subscription strategy σ_{-i}, computed using Bayes' rule (see subsubsection B.4.1 of the Appendix).

Next is the subscription stage, in which we assume that a customer subscribes iff this increases her expected future payoff (or alternatively that the frictional cost to subscribing is small). Specifically, the best-response subscription strategy of customer i is a function of her valuation for the service during the slow period $v_{s,i}$, of the announced deal price r_s and of the threshold n, as well as the vectors of subscription and visit strategies of all other players, σ_{-i}, ν_{-i}. The subscription stage belongs to the class of Coordination games first defined by Schelling (1960): in this type of game there are typically multiple equilibrium outcomes, where if enough customers coordinate on a certain decision, a single customer has no incentives to deviate from what the majority does. The multiple Perfect Bayesian Equilibria that arise can be meaningfully grouped into two types. In type I equilibria, customers subscribe if their valuation for the slow period is sufficiently high, while in type II equilibria, customers never subscribe to the deal: as a consequence, the deal is never active, and therefore not subscribing is optimal. We discard type II equilibria from our analysis for two reasons. First, because we show that a type I equilibrium Pareto Dominates a type II equilibrium; that is, customers are better off coordinating on a type I equilibrium than on a type II equilibrium.

The reason is that by subscribing to the deal, they have a chance to get a discount and visit in their preferred slow period, while at the same time increasing availability for those who did not subscribe (see Appendix, subsubsection B.4.6). The second reason is that type II equilibria do not exist if we consider the more general case of a consumer population where only a fraction of customers are strategic, as in our extension in Section 10.3. Therefore, we restrict our attention hereafter to type I equilibria, which we characterize in the next lemma.

Lemma 3. *Equilibrium strategies in Customer Continuation Game*

In a type I equilibrium:

(1) *A customer subscribes to the deal iff her valuation for the service in the slow period is higher than a certain threshold.*

(2) *A customer visits during the slow period iff the deal is active and her valuation is higher than a threshold; she visits during the hot period otherwise.*

(3) *The subscription and the visit thresholds coincide.*

In equilibrium, customer subscription and visit strategies are of a threshold type, and the thresholds for the two strategies coincide, since customers who would visit in the slow period are the same as those who subscribe to the deal. Thus, customer behavior can be fully summarized by just one threshold, $\hat{v}_t$, such that a customer with a slow-period valuation lower than $\hat{v}_t$ does not subscribe to the offer and visits in the hot period, whereas a customer with a slow-period valuation higher than $\hat{v}_t$ subscribes to the offer, and then visits in the slow period if the deal is active and in the hot period when the deal is not active. The threshold valuation $\hat{v}_t$ is the one that, conditional on the deal being active, makes a customer indifferent between visiting in the slow and in the hot period–since when the deal is not active both subscribers and non-subscribers visit on the hot period and earn the same surplus. The following equation compares the threshold customer's surplus in each period, when the deal is active, for any deal offer (r_s, n):

$$(\hat{v}_t - r_s) \int\limits_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty} \min\left(1, \frac{k}{\bar{H}\left(\hat{v}_t\right) x}\right) \mathrm{d}G_c\left(x\right) = (v_h - r_h) \int\limits_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty} \min\left(1, \frac{k}{H\left(\hat{v}_t\right) x}\right) \mathrm{d}G_c\left(x\right). \tag{10.1}$$

The LHS represents customer surplus when she visits in the slow period, and the RHS when she visits in the hot period. Unfortunately, for a general deal offer (r_s, n) there can be multiple solutions to (10.1), and consequently multiple type I equilibria. An increase in the threshold, $\hat{v}_t$, implies an increase in the number of visitors in the hot period and a corresponding decrease in the number of visitors in the slow period. While the hot period surplus decreases when $\hat{v}_t$ increases because a higher fraction of customers visiting in the hot period reduces availability, the slow period surplus generally does not increase in $\hat{v}_t$. A higher threshold $\hat{v}_t$ implies fewer customers visiting in the slow period, which should increase availability (the visit effect), but it also means that a smaller fraction of customers subscribe to the deal, which implies that the deal is active only when demand is higher, which in turn implies lower availability (the subscription effect). The overall effect is therefore ambiguous.

However, we can show that there exists a unique solution to (10.1) when the announced price of the deal, r_s, is higher than a certain level $\bar{r}$. To understand the drivers of this effect, it is instructive to rewrite (10.1) in terms of a comparison between relative availability and relative surplus in the two periods:

(10.2)
$$\frac{\hat{v}_t - r_s}{v_h - r_h} = \frac{\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty} \min\left(1, k\left(H\left(\hat{v}_t\right)x\right)^{-1}\right) \mathrm{d}G_c(x)}{\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty} \min\left(1, k\left(\bar{H}\left(\hat{v}_t\right)x\right)^{-1}\right) \mathrm{d}G_c(x)}.$$

The LHS of the rewritten equation is the ratio of the service surplus in the slow period to that in the hot period, whereas the RHS is the ratio of service availability in the hot period to that in the slow period. The ratio of the service surplus (LHS) is always increasing in the threshold, $\hat{v}_t$. When the deal price r_s is higher than $\bar{r} = H^{-1}\left(\frac{1}{2}\right) - v_h + r_h$, a higher fraction of customers visit in the hot period, i.e. $H\left(\hat{v}_t\right) \geq \frac{1}{2}$, which ensures that the ratio of service availability always decreases in the customer threshold, $\hat{v}_t$. To see why, note that, as before, a higher threshold implies a smaller fraction of visitors in the slow period and a higher fraction in the hot period, thus decreasing the service availability ratio (the visit effect). Also as before, a higher $\hat{v}_t$ implies fewer subscribers, which means that the deal is active only when demand is higher (the subscription effect): however, since $r_s \geq \bar{r} \iff H\left(\hat{v}_t\right) \geq \frac{1}{2}$, this implies that the impact of higher demand in the hot period is more severe than in the slow period. Hence, a price $r_s \geq \bar{r}$ ensures that there exists a unique equilibrium for the customer continuation game. We will show that this is always the case for the full game.

Firm Optimal Announcement and Equilibrium Outcome. The service provider chooses the slow period price r_s and the activation threshold n that maximize expected profit, taking into account customer best-response strategy $\hat{v}_t\left(r_s, n\right)$ characterized in (10.1). The expected profit of the firm is then

$$\Pi_t = \max_{r_s,n} \left[(r_h - c) \int_0^{\frac{n}{\bar{\alpha}_t}} (\min(k,x) - c_F)\,\mathrm{d}G(x) + \int_{\frac{n}{\bar{\alpha}_t}}^{+\infty} (\min(k,\alpha_t x)(r_h - c) + \min(k,\bar{\alpha}_t x)(r_s - c) - 2c_F)\,\mathrm{d}G(x) \right]$$

$$s.t.\, r_s < r_h,\, n > 0,$$

where $\alpha_t = H(\hat{v}_t(r_s, n))$ and $\bar{\alpha}_t = \bar{H}(\hat{v}_t(r_s, n))$ are the fractions of customers that visit during the hot and slow periods, respectively, when the firm announces the deal (r_s, n).

Lemma 4. *The firm can restrict to deal offers with a discounted price higher than $\bar{r}$ without any reduction in his expected profit.*

This result states that the firm needs to consider only announcements with a discounted price higher than $\bar{r}$, because it is never optimal to discount so much that more than half of the customers visit in the slow period when the deal is active. This Lemma shows that even though there might be multiple type I equilibria in the customer continuation game that ensues after the deal is announced, there is a unique equilibrium for the full game, because the firm is always better off announcing a deal for which there exists a unique customer best response $\hat{v}_t(r_s, n)$. We next compare the profits under the unique equilibrium outcome of the threshold discounting game with those from the traditional approach, that is, with the highest profit between closure and price discrimination.

10.2. **Comparing Threshold Discounting with the Traditional Approach.**

Theorem 7. *Threshold discounting outperforms the traditional approach, i.e. $\Pi_t > \Pi_a$.*

The superior performance of threshold discounting arises from its most characteristic feature, i.e., the activation threshold, which gives rise to two independent sources of advantage: a responsive duality effect and a strategic scarcity effect.

Responsive Duality. Lemma 2 showed that closing in the slow period ends up earning a higher profit than opening and discounting if and only if market size is below a threshold. Unfortunately, a firm considering the traditional approach needs to decide whether to employ price discrimination or seasonal closure ex-ante, without knowing the market state, and the choice that maximizes the expected profit may turn out to be wrong in retrospect once market size is realized and customers visit the firm. With threshold discounting, the firm does not have to trade off the relative strengths of seasonal closure and price discrimination, because he can get the best of both worlds.

An appropriately designed threshold discounting offer allows the firm to ensure that the deal gets activated only in those states of the world in which the market size turns out to be above a threshold of his choice (Appendix, Section B.8, Lemma 8). In such a contingency, the firm is effectively imitating the demand-balancing effect of a traditional price discrimination approach. On the other hand, when the market size is below this threshold, the deal is not activated and the service is not offered in the slow period, so that the firm achieves fixed-cost optimization and full margins by effectively using the seasonal closure approach. Taken together, the activation threshold endows threshold discounting with *a responsive duality*, i.e. a built-in, market-responsive dual mechanism that allows the firm to use the information supplied by customers to determine what demand manipulation technique to employ, an advantage unavailable with the traditional approach.

This responsive duality is not the only advantage of threshold discounting, the benefits go further. Even more interesting is a strategic scarcity effect created by threshold discounting, which allows a firm to better price discriminate strategic customers than does the traditional price discrimination approach, thus improving capacity utilization even further.

Strategic Scarcity Effect. Customers strategically think about price and availability and they react differently to a slow-period discount that is active contingent on high market size, as opposed to a discount that is always active. In particular, we find that a discount conditional on a high enough market size increases the fraction of demand diverted from the hot to the slow period, as compared to the same level of a non-contingent discount. Formally, we find that for any activation threshold $n > 0$ and discounted price $r \geq \bar{r}$ we have that $H\left(\hat{v}_t\left(r, n\right)\right) < H\left(\hat{v}_p\left(r\right)\right)$ (subsubsection B.7.1 of the Appendix). We call this observation the *strategic scarcity effect.*

Strategic scarcity is beneficial because it accomplishes the same result as would a price reduction, that is, attracting more customers during the slow period to achieve a more equitable allocation of demand across periods, but it comes as a free lunch, i.e., the provider enjoys the additional reallocation of demand without paying through higher discounts or lost margins. Put differently, strategic scarcity is beneficial because it magnifies the returns from any discounting level by increasing strategic customers' elasticity to price reductions compared to the traditional price discrimination approach.

The key cause of this effect lies in the difference in service availability between the hot and slow periods under the two discount schemes. Under threshold discounting, the fact that the deal is active signals to the customers that the market size is high enough. This implies that availability will be lower in both time periods, but *more so in the hot period*, making the slow period more desirable to customers.[23] Thus, a customer who is indifferent between the two periods under normal price discrimination is instead willing to visit during the slow period under threshold discounting when the deal is active, because the active deal signals that the market size is higher than average, hence the odds of being served shift further in favor of the slow period. Overall, the higher effectiveness of threshold discounting due to strategic scarcity effectively implies that the service provider can achieve the same level of expected capacity utilization that a price discrimination strategy would, while keeping the expected unit margin higher.

A novel operational advantage. To summarize, the advantages of threshold discounting stem from 1) its responsive dual nature, imitating the fixed cost savings of the seasonal closure approach when market size is low and the demand-balancing effect of the price discrimination approach when market size is high; and 2) increasing customer responsiveness to slow-period discounts by unraveling market size, which enables the customer to use this information in estimating service availability and self-selecting the consumption period, thus increasing capacity utilization for any discount level offered.

In essence, a threshold discounting offer acts as a coordination mechanism: it insures that customers are better off participating, but it also insures that their participation ultimately benefits the firm. This coordination is notable because when it comes to pricing decisions, getting buyers and sellers to agree is difficult, often leading to zero-sum games in which making both parties better off is not possible. Such beneficial coordination is possible in this case because of three synergic factors: the discount encourages customers to subscribe, the activation threshold insures a "responsive discount" that triggers when market size is high enough, and the firm is better off offering discounts when the market size is high (as the need to smooth demand arises).[24] To see why ensuring customers participation and employing a "responsive discounting" scheme is not necessarily beneficial, consider the opposite case in which the firm offers a discount when the threshold is *not* met. This would still encourage customers to subscribe, and it would still allow the firm to respond

[23]A simple example can clarify this property: suppose that capacity is 10, that 60% of customers visit in the hot period, and that the market size is either 10, 20 or 30 with equal odds; then the expected availability of the hot period relative to the slow period is $\frac{1+10/12+10/18}{1+1+10/12} = \frac{43}{51} \simeq 0.84$ over all market states, $\frac{10/12+10/18}{1+10/12} = \frac{25}{33} \simeq 0.76$ over the two higher states, and $\frac{10/18}{10/12} = \frac{2}{3} \simeq 0.67$ for the highest state; that is, the expected service availability of the hot period relative to the slow period decreases as we consider only increasingly higher states–as strategic customers do when they learn that the activation threshold has been reached.

[24]This coordination is still possible but more difficult to achieve if the distribution of customer preferences is itself uncertain. These additional results are available from the authors upon request.

to different market conditions with different pricing; however, this "responsive discounting" would strictly reduce the profit of the firm, even in the absence of fixed costs, because is would increase sales cannibalization when the market size is lower, and it would not improve demand smoothing when market size is higher–that is, when this is needed (subsubsection B.8.1).[25]

10.3. **A Mixed Population of Strategic and Myopic Customers.** The above analysis has so far assumed that all customers are strategic, in the sense that they all account for other customers' subscription and visit responses to the discounting scheme offered by the firm when they make their decisions. Arguably, not all customers are sophisticated enough to do this: Li et al. (2014), for example, empirically estimate the percentage of strategic consumers in the airline industry to be between 5.2% and 19.2%. In this section, we extend our analysis to consider a mixed population in which a fraction γ of customers are strategic, and the remaining fraction $1-\gamma$ are myopic, in that they do not account for the decisions of other customers. This means that in making her decision, a myopic customer naively ignores both the odds of the deal being active and the expected availability in each period, since these depend respectively on the subscription and visit strategies of the other customers. This customer subscribes/visits in the slow period iff her service surplus is higher than in the hot period, i.e. iff $v_s - r_s > v_h - r_h$, where v_s is her slow-period valuation. The profit of threshold discounting when only a fraction γ of the population is strategic is given by

$$\Pi_t^{\gamma} = \max_{r_s,n}\Bigg[(r_h - c)\int_0^{n\,\bar{\alpha}_{t,\gamma}(r_s,n)^{-1}} (\min(k,x) - c_F)\;\mathrm{d}G(x) + \tag{10.3}$$

$$+\int_{n\,\bar{\alpha}_{t,\gamma}(r_s,n)^{-1}}^{+\infty} (\min(k,\alpha_{t,\gamma}(r_s,n)\,x)(r_h - c) + \min(k,\bar{\alpha}_{t,\gamma}(r_s,n)\,x)(r_s - c) - 2c_F)\;\mathrm{d}G(x)\Bigg]$$

$$s.t.\; r_s < r_h,\; n > 0,$$

where $\alpha_{t,\gamma} = \gamma H\left(\hat{v}_t\left(r_s,n,\gamma\right)\right) + (1-\gamma)\,H\left(v_h - r_h + r_s\right)$ is the fraction of the population that in equilibrium visits the firm during the slow period when the deal is active, $\bar{\alpha}_{t,\gamma} = 1 - \alpha_{t,\gamma}$, and

[25]As pointed out in the introduction, anecdotal popular press discussions of the use of threshold discounting have focused on their network effects and a consequent demand increase. Note that our model deliberately leaves out network effects to focus on operational performance, and our effects stem solely from the better demand-supply matching enabled by threshold discounting. Further, all the results presented above continue to hold for $c_F = 0$, that is, even when there are *no economies of scale*. This suggests that these innovative and profit-enhancing schemes do not need to be the exclusive prerogative of high-volume businesses, but rather can be employed by small businesses–such as those featured by Groupon and its competitors–with equally beneficial results.

where $\hat{v}_t(r_s, n, \gamma)$ is defined as in (10.2), with $H(\hat{v}_t)$ and $\bar{H}(\hat{v}_t)$ being replaced by $\alpha_{t,\gamma}$ and $\bar{\alpha}_{t,\gamma}$ respectively.

Theorem 8. *With a mixed population of customers, threshold discounting still outperforms the traditional approach, i.e. both price discrimination and seasonal closure. This is true even when the population is entirely myopic.*

As explained above, the advantage of threshold discounting is driven both by its ability to mimic closure and price discrimination when most appropriate, as well as from the strategic scarcity effect it creates. While the strategic scarcity effect relies on customers' ability to account for the decisions of other customers when making their decisions, the responsive duality advantage exploits the information contained in the number of subscribers that does *not* require customers to be strategic, but rather to signal if they are planning to visit during the slow period. Thus, even when there are no strategic customers in the population, the operational advantages of threshold discounting persist.

Next, we study the impact that the proportion of strategic customers in the population has on the profits of a service provider employing threshold discounting. Most of the existing literature on strategic customers (Su and Zhang (2008); Liu and van Ryzin (2008); Cachon and Swinney (2009, 2011)) has either proven that strategic customers are a threat to a firm's profit, or has taken it as a given and developed countermeasures to reduce their negative effect.[26] The typical setting often evoked is one in which an apparel retailer sells a finite inventory over a finite season, and may resort to price markdowns at the end of the season to dispose of leftover inventory. By anticipating price markdowns, strategic customers can decide to postpone their purchases until the end of the season, thus reducing profits for the firm. Our setting shares many characteristics with this typical setting. In Cachon and Swinney (2009), for example, strategic customers can decide to purchase in two different periods–during the season, when their valuation for the product is higher, or at the end of the season, when their valuation is lower–which maps exactly to the hot and slow periods in our framework. As in our paper, in Cachon and Swinney (2009) the firm offers a reduced price in the period that customers value the least. Finally, as in our paper, strategic customers take into account the actions of other customers and act to maximize their expected surplus. Despite these

[26] A rare exception is the empirical work by Li et al. (2014), which argues that if, on the one hand, strategic customers reduce margins, they on the other increase demand, either by forcing the firm to reduce prices, which in itself raises demand, or by postponing purchases and thus having a second purchasing opportunity. Thus, the effect on profit may go either way. As will soon be clear, our result differs from theirs, and it stems from a different reason.

similarities, the effects of strategic customers in our setting are in stark contrast with those in the classic settings studied in the literature.

Theorem 9. *The profits under threshold discounting are higher with more strategic customers in the population. Formally, if $\gamma_2 > \gamma_1$, then $\Pi_t^{\gamma_2} > \Pi_t^{\gamma_1}$.*

Strategic customers differ from myopic ones in that, by accounting for the actions of the other players, they can better account for future prices and availability, and act accordingly. In the classic setting, this leads strategic customers to wait for otherwise unanticipated price markdowns, and this is always harmful for the firm. In our setting, strategic behavior has different implications. First, strategic customers account for the visit decision of the other customers, which allows them to form expectations on the service availability of each period, accounting for the odds of getting a unit of service before they visit, which is in the interest of the firm. Second, they also account for the subscription decision of other customers, which allows them to refine their expectation on service availability upon knowing that the deal is active (strategic scarcity effect) which also goes in the interest of the firm, as already discussed. In our context, there is no difference in how strategic and myopic customers account for price reductions, since the firm clearly announces them upfront before the subscription stage–and with good reason, as discussed below. Hence, the sophisticated decision process of strategic customers always has a beneficial impact for the firm.

It should be noted that the firm's initial commitment to a price reduction has nothing to do with the use of price commitment strategies as a countermeasure to strategic customers, as for example studied in Su and Zhang (2008). In their setting, the firm commits to high enough prices at the end of the season to induce strategic customers to purchase in-season, i.e., in the "hot" period. In our setting, the firm announces price reductions to achieve the opposite effect, i.e., redirect customers from the hot period into the slow period. The difference arises because they consider a firm selling inventory of a durable good, while we consider a service firm selling capacity. For a firm selling a physical product, a customer who decides to purchase in the low season rather than in the high season is always harmful, because it reduces margins: hence, the firm commits to high prices in the low season to prevent such behavior from occurring. For a service firm selling capacity, a customer who decides to purchase in the slow period rather than in the hot period may instead be beneficial, because it increases sales whenever capacity in the hot period is sold out, but there is still spare capacity in the slow period: hence, the firm commits to (appropriate) price reductions in the slow period to incentivize such behavior. Basically, the perishable nature of capacity transforms strategic customer's inter-temporal purchasing decisions from a threat to margins into an opportunity to increase capacity utilization and sales, which is why the firm is better off announcing price reductions to all customers.

11. Design Considerations in Threshold Discounting Offers

The above analysis examined one particular design of threshold discounting, that in which the firm pre-commits to the activation threshold for the deal, if the deal is active or not is announced before the beginning of both time periods, and the discount can be used only during the slow period. In practice, we encountered numerous variations of this basic setup and, at different points of time, Groupon experimented with other arrangements. Hence, in this section we examine alternative designs and compare them with the original design in Section 10, henceforth referred to as *classic* threshold discounting.

11.1. **Opaque Activation Rule.**

In classic threshold discounting, the firm commits to a discounted price and an activation threshold before customers make the decision to subscribe or not: this commitment ties the service provider's hands, forcing him to abide by a specific activation rule. A potentially better design is one in which the provider does not publicly commit to a decision rule for activation, and instead makes the activation decision after he observes how many customers have subscribed: it is in fact well-known that postponing a decision to a later time is beneficial if this allows the acquisition of new information that is relevant for that decision–as in this case, where subscriptions contain new information on the market state, which is relevant to making the activation decision. With such a design (Figure 11.1) the service provider announces the discount price r_o before the customers' subscription decision, yet does not commit to any activation rule. After customers have subscribed, the provider observes the number of subscribers, and only then announces whether or not the deal is active. Designs of this kind are quite common in Customer Voting Systems, whereby customers may be asked to vote for new product designs that could be developed by the firm in the near future, but the firm does not commit to any specific development rule in advance.[27]

[27] See **?** and the customer voting system at Modcloth.

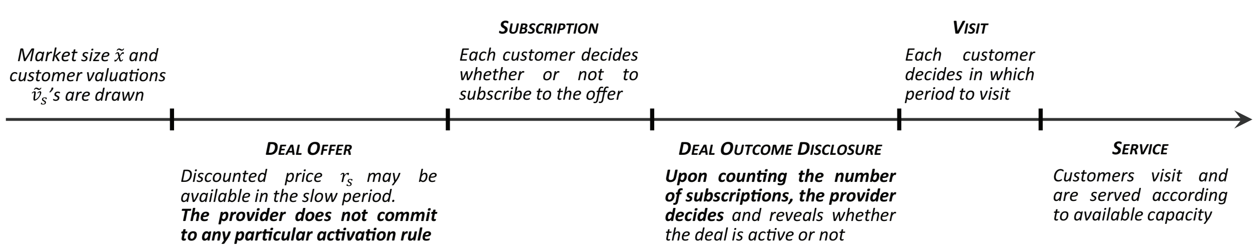


Figure 11.1: Timeline of Threshold Discounting with Opaque Activation Rule

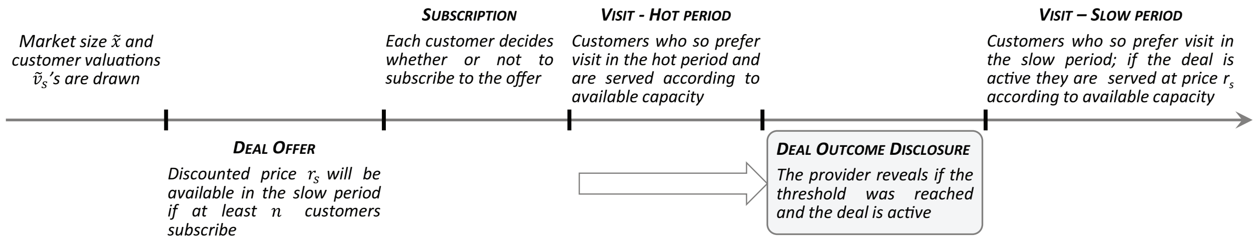


FIGURE 11.2: Timeline of Threshold Discounting with Late disclosure

However, postponing the activation decision to a later time may have its drawbacks, because committing to an activation threshold can give the firm a strategic first-mover advantage over the customers. Hence, the benefit of an opaque activation rule will depend on the relative strengths of the informational advantage of postponement on the one hand, and the strategic advantage of commitment on the other. The next theorem compares the two designs.

Theorem 10. *Offering threshold discounts with a committed threshold-activation rule, as in the classic threshold discounting, is better for the firm than offering discounts with an opaque activation rule.*

As conjectured, by not committing to a specific threshold in advance the firm loses the strategic advantage of being able to use the activation threshold to "steer" customers towards the desired equilibrium. Unexpectedly, however, postponing the activation decision does not provide the firm with any informational advantage, despite the fact that this allows the firm to acquire new information that is relevant to making the deal activation decision. The reason for this unexpected result is that, though relevant, the information contained in the subscriptions always leads to the optimal activation rule being a threshold decision, which the firm can determine already with the information available before the customer subscription stage. Hence, committing to a threshold activation rule upfront provides strategic benefits and no informational disadvantage, and a classic threshold discounting outperforms one with an opaque activation rule.[28]

11.2. **Time when the Outcome of the Deal is announced.** In classic threshold discounting, the service provider releases information about the outcome of the deal, i.e., whether or not it is active, before both time periods begin, allowing customers to make a consumption decision knowing

[28]There can be cases in which some additional relevant information is exogenously revealed between the time the deal is announced and the time subscriptions are closed, as uncertainty over weather conditions in the case of an outdoor performance: in these cases, postponing the activation rule may lead to an informational advantage, and which design is better depends on the relative strength of the benefits of commitment and those of postponement.

if the threshold has been reached. However, in cases in which there is enough time between the hot and slow periods, the service provider could decide to disclose such information after the hot period is over but before the slow period begins.[29] Strategic customers are responsive to price reductions, but also to changes in perceived availability. Liu and van Ryzin (2008) and Yin et al. (2009) have shown how a firm dealing with strategic customers can benefit from increasing the rationing risk they perceive. It is therefore important to study the impact of postponing the deal outcome revelation to customers, since doing so increases the uncertainty–hence the risk–of their subsequent visit decisions, and could therefore lead to a similar effect. The sequence of decisions and information revelation is described in Figure 11.2. As in classic threshold discounting, the terms of the deal–the discount and the activation threshold–are announced upfront. The only difference is that the outcome disclosure stage now follows the hot period, whereas in the original model it preceded both the hot and slow periods.

Under late disclosure, the profit of the service provider takes the form

$$\Pi_l = \max_{r_s,n}\left[(r_h-c)\int_0^{\frac{n}{\bar{\alpha}_l}}(\min(k,\alpha_l x)-c_F)\mathrm{d}G(x)+\int_{\frac{n}{\bar{\alpha}_l}}^{+\infty}(\min(k,\alpha_l x)(r_h-c)+\min(k,\bar{\alpha}_l x)(r_s-c)-2c_F)\mathrm{d}G(x)\right]$$

$$s.t.\, r_s < r_h,\, n > 0,$$

where $\bar{\alpha}_l$ and α_l are the fractions of subscribers and non-subscribers, and are a function of the deal discount, r_s, and activation threshold, n.

Theorem 11. *Classic threshold discounting, i.e. with early disclosure, achieves a higher profit for the firm than threshold discounting with late disclosure.*

Unlike in other similar settings, inducing a rationing risk on strategic customers–by postponing the disclosure decision–turns out to be unwise. Late disclosure of the deal outcome has two main implications for the firm. First, it impairs the inter-temporal substitutability of demand. Specifically, in the event that the deal is not active, the firm loses sales to those customers who subscribed to the deal and did not visit the service provider during the hot period because they intended to visit during the slow period. The second implication is a consequence of the first, and it is of a strategic nature. Given that subscribing to the deal and waiting for the slow period does not guarantee that the provider will be open at that time, strategic customers are less willing to visit during

[29] If the slow period comes before the hot period, this scheme is obviously not viable.

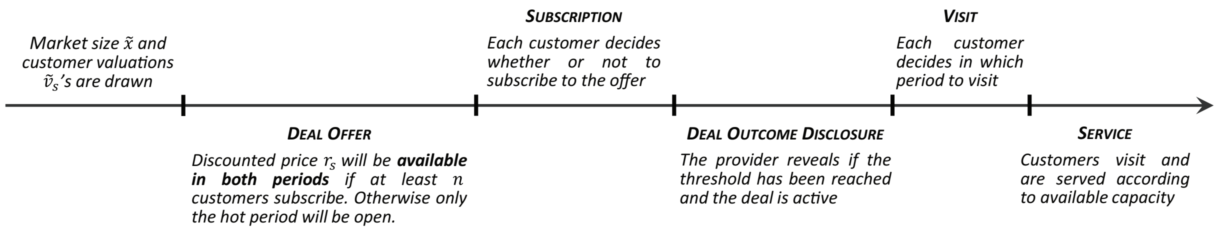


FIGURE 11.3: Timeline for Unrestricted Threshold Discounting

the slow time period than in the case of early disclosure, i.e. $\alpha_l(r,n) > \alpha_t(r,n)$ for every price $r \geq \bar{r}$ and every activation threshold $n > 0$. This has negative implications for profit, because the service provider needs to offer customers a higher discount for them to visit during the slow time period, further reducing margins. Basically, this strategic implication works in the opposite way of the strategic scarcity effect described in the discussion of Theorem 7, reducing the effectiveness of discounts as inter-temporal demand-balancing devices.

Taken together, the previous results show that providing customers with a transparent activation rule and full and timely information on the activation of the deal makes threshold discounting schemes most potent, or put differently, the less the uncertainty on the customer side, the more effective threshold discounting becomes at increasing capacity utilization and profit.

11.3. **Time Restricted Discounts.** While classic threshold discounting restricts the use of the discount to slow periods, discounted offers featured by Groupon and its numerous copycats often place no constraints on the time period of service, i.e., if activated, the discount can be used during hot and slow periods alike. The timeline for these type of deals, henceforth named unrestricted threshold discounting (subscript u), is otherwise the same as for classical threshold discounting, and it is described in Figure 11.3.

Theorem 12. *Classic threshold discounting achieves a strictly higher profit for the firm than unrestricted threshold discounting.*

Classic threshold discounting is strictly better than unrestricted threshold discounting: by allowing customers to enjoy a reduced price in any period of their choice, the service provider cripples the main advantage of price reductions, that is, the ability to price discriminate between the hot and slow periods in order to redirect some demand to the latter and improve capacity utilization. Despite charging the same price in both periods, unrestricted discounting can still redirect some demand to the slow period; in fact, a price reduction increases the service surplus in both periods, increasing the surplus loss for a customer from not obtaining a unit of service, thus making customers more willing to visit in the slow period where availability is higher. However, the magnitude of this demand-balancing effect is small compared to what can be achieved using price discrimination. Moreover, the cost associated with a price reduction under unrestricted threshold discounting is much higher than under classic threshold discounting, as the service provider reduces his margin in both time periods. As a result, under unrestricted threshold discounting, price reductions come at a higher cost and yield a smaller operational benefit than classic threshold discounting.

While the overall benefit of unrestricted threshold discounting will ultimately depend on the sum of many effects (see for example Edelman et al. (2011)), from a purely operational point of view this design has severely unattractive features, and in many cases a service provider would be better off simply using the traditional approaches. In our numerical study (Section 12) we find that a traditional approach is better than unrestricted threshold discounting in 90% of our scenarios, resulting on average in a 2% higher profit. This analysis may help explain the oft-repeated assertion that Groupon-like deals were worse for many businesses than just following the traditional approach to managing demand and capacity.[30] Perhaps the wide use of unrestricted discounting has been a consequence of the type of contracts used in the industry when a service providers channels its offer through powerful intermediaries such as Groupon. Such contracts reward the intermediary with a fraction of the revenues channeled through the deal: under unrestricted threshold discounting the amount of revenues earned on subscribers is substantially higher compared to classic threshold discounting–and so is the commission earned by the intermediary.

11.4. **Focused Threshold Discounting.** One way to potentially improve threshold discounting is to observe that not all customers need to be incentivized to visit during the slow period. Customers with a high enough slow-period valuation value the hot period almost as much as the slow period and prefer to visit the firm during the slow period even when no discount is offered due to higher service availability. If we let n^t be the equilibrium activation threshold in classic threshold discounting, then this is true for $v_s \geq \hat{v}_t\left(r_h, n^t\right)$. This means that classic threshold discounting is inefficient, in that it ends up providing unnecessary monetary incentives to these customers, a source of inefficiency that could be remedied by focusing the incentives on those customers who actually need them. Next, we explain the intuition behind focused threshold discounting, and then study it formally.

[30]See, for example, "Groupon in retrospect", poesie's cafe blog, September 2010, http://goo.gl/R4lJW

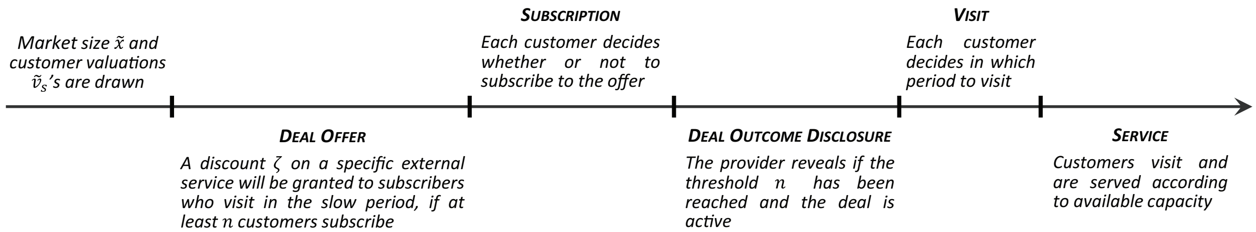


FIGURE 11.4: Timeline of a Focused Threshold Discounting

Consider an opera house performing *Rigoletto* on Saturday and Sunday nights. Potential customers are comprised of active workers, who prefer Saturday over Sunday–albeit with different degrees of preference–and retired workers, who don't care about time and therefore prefer Sunday evening due to higher availability. Consider a service desired by active workers but not desired by retired workers, such as baby-sitting. Then a focused threshold discount that offers free baby-sitting service to subscribers for the Sunday night show could redirect the desired number of customers to Sunday without offering unnecessary discounts to retired workers, thus improving profits.

To formalize this intuition, suppose that customers can be divided into two segments, one characterized by strong time preferences ($v_s < \bar{v}_s$) that attach to the external service a positive value $V > 0$, and another with weak time preferences ($v_s \geq \bar{v}_s$) that find no value in the service, with the frontier valuation $\bar{v}_s$ being high enough. Specifically, let v_e be the value attached by a customer to an external service, where

$$v_e = \begin{cases} V & if\, v_s < \bar{v}_s \\ 0 & otherwise \end{cases} \quad with \quad \bar{v}_s \in \left[\hat{v}_t\left(r_h, n^t\right), v_h - \epsilon\right], \forall \epsilon > 0. \tag{11.1}$$

Focused threshold discounting consists of promising subscribers not a discounted price, but rather a discount on the external service if they visit during the slow period (Figure 11.4).

Theorem 13. *Under the conditions in (11.1), focused threshold discounting improves profit for the firm compared to classic threshold discounting.*

Indeed, when customers can be segmented in a way that links their time preferences to their interest for some other service, the firm can employ focused threshold discounting to improve the efficiency of his incentive system, thus achieving a higher profit.

12. Numerical Study

In this section, we present the results of a numerical study that helps us illustrate the advantages of classic threshold discounting, as presented in Section 10. We consider the usage of threshold discounting at a potential service provider, the opera house Teatro Regio located in Torino, Italy. We extrapolate cost data from their 2011 balance sheet, and we use their pricing data to guide our choice for customers' inter-temporal preference parameters. Table 12.1 illustrates the values chosen for each parameter and the criteria employed. In the absence of complete data on customer preferences and cost structure, we consider multiple alternate choices whenever we are not fully certain of the obtained parameters. Specifically, we consider five different market size distributions, twelve different preference distributions, and five different cost structures–the actual cost structure of Teatro Regio, plus four additional scenarios–and we simulate all possible combinations of these parameters, for a total of 300 scenarios examined.

Figure 12.1 (a) shows the profit gains of threshold discounting over the traditional approach (i.e. the best between closure and price discrimination) for each of the 300 scenarios simulated. Profit gains can be substantial, up to about 29% (average 6.1%), which is quite remarkable if one considers that the cost of implementing threshold discounting is negligible. Figure 12.1 (b) and (c) show how profit gains change as market uncertainty increases while the mean market size remains constant, for different levels of fixed costs (b) and customer preferences (c). Interestingly, profit gains are higher when market uncertainty increases, suggesting that threshold discounting is particularly useful in the presence of high market uncertainty, with the highest gains accruing when fixed costs are at an intermediate level (since the ex-ante choice between seasonal closure and price discrimination can turn out to be very costly, hence responsive duality is most effective) and when customer preferences for the hot over the slow period are weaker (since price discrimination is more effective than closure, and strategic effects make threshold discounting more potent at price discriminating). A finer search reveals that profit gains actually increase in market uncertainty in all 300 scenarios considered.

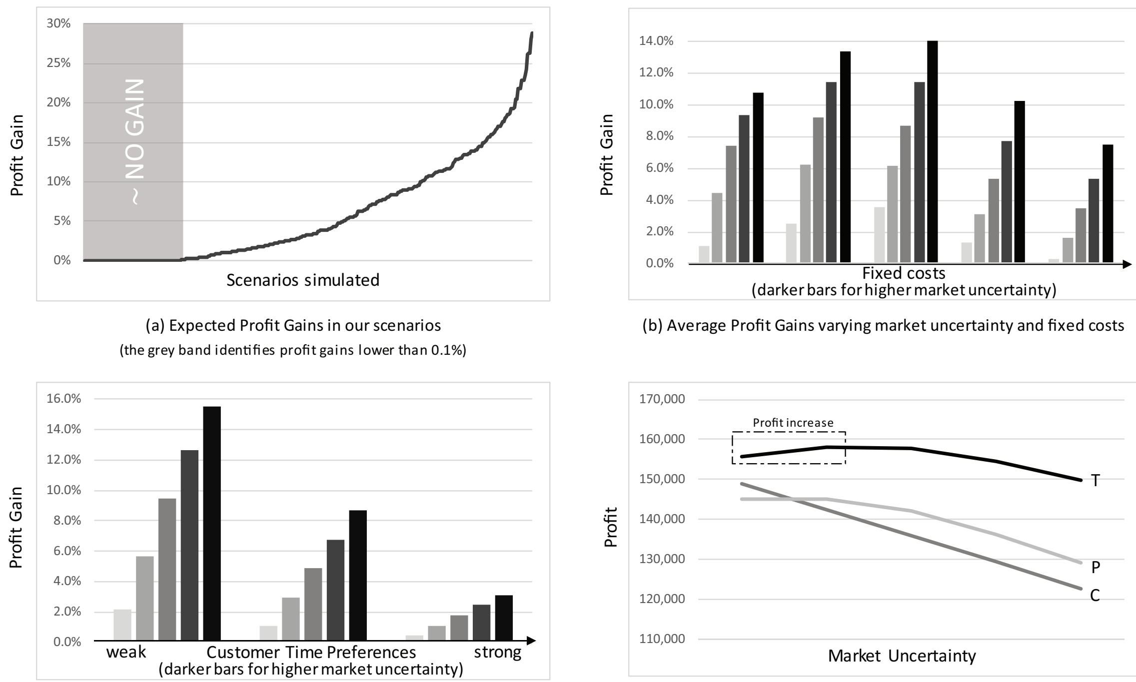
Figure 12.1 (d) shows the impact of market uncertainty on the profit of threshold discounting, price discrimination, and closure for a representative set of parameters. Note that higher uncertainty reduces the profit of both closure and price discrimination, as one would expect given the concavity of profit with respect to market size realization (Figure 9.2). However, higher market uncertainty is much less of a threat for a firm using threshold discounting, as it reduces profit to a lower extent or, contrary to intuition, it may even *increase* profit, as in the left part of (d). This is because

Parameter	*Value(s) Considered*	*Source*
Capacity, k	1500	The actual capacity of Teatro Regio is 1582 seats, of which 1530 are proper seats (the rest being stools). We rounded down to 1500
Market size, $\tilde{x}$	Uniform distribution, mean 2250 ($=1.5k$) and alternate widths 900, 1500, 2100, 2700, 3300	Given the popularity of the Teatro Regio in the last three years, we consider the average market over two periods to be 1.5 times the single-period capacity. We have no information on demand variability, thus we use different mean-preserving spreads to study the impact of market uncertainty on performance measures.
Fixed cost, c_F	k€ 35, 50, 70, 90, 105	The fixed costs that could be saved by closing down on a given night at Teatro Regio are estimated to be about 70K€, which comprises the per-show payroll for external performers and the cost of utilities. We also analyze a broader set of values in order to study the impact of fixed costs on performance metrics of interest.
Full price, r_h	€ 130	The price charged for prime-time performances during the season (if we exclude the day of inauguration).
Upper valuation, v_h	€ 140, 170	We examine two potential hot-period valuations.
Lower valuation, $\underline{v}$	€ 0, 40	We examine two cases, € 40 the lowest price charged at Teatro Regio for "slow" periods, € 0 the extreme case such that a customers obtains no value from attending the event.
Inter-temporal preferences, $\tilde{v}_s$	Beta distribution with support $[\underline{v}, v_h)$ and shape parameters $(1,1)$, $(1,2)$, and $(2,1)$	We examine three different scenarios, going from one in which most customers have a strong preference for the hot period (1,2) to one in which most customers have only a mild preference for the hot period (2,1), with (1,1) being an intermediate case.
Unit cost, c	€ 0	The marginal cost of issuing a ticket is negligible.
Fraction of strategic customers, γ	10%	A conservative average of the empirical findings in Li et al. (2014).

TABLE 12.1: Parameter values employed in the numerical analysis

higher market uncertainty amplifies the responsive duality advantage of threshold discounting, and this effect may offset the negative effects traditionally associated with market uncertainty.

Another worthy observation is that, under all scenarios, threshold discounting sets a higher discount compared to price discrimination (42.5% vs 34.8% on average, respectively). A firm employing



(a) Expected Profit Gains in our scenarios
(the grey band identifies profit gains lower than 0.1%)

(b) Average Profit Gains varying market uncertainty and fixed costs

(c) Average Profit Gains varying market uncertainty and customer preferences
(beta shape parameters equal to (2,1),(1,1),(1,2) from weak to strong preferences)

(d) Expected Profit as a function of market uncertainty
(T= Threshold Discounting, P= Price Discrimination, C= Closure)

FIGURE 12.1: Profit gains of Threshold Discounting over Traditional Approaches; 300 scenarios simulated

Profit gains are measured as the percentage increase in profit when employing threshold discounting compared to the best between price discrimination and closure. Market uncertainty refers to the standard deviation of the market size distribution, G.
Figure d: the instance represented has $c_F = 50000$, $v_h = 140$, $\underline{v} = 0$, $(a, b) = (1, 1)$.

price discrimination must be cautious with discounts because they reduce profit in low market states by reducing margins without increasing sales. A firm employing threshold discounting, on the contrary, offers the discount only when the market state is high enough and the deal becomes active, and can therefore exploit the full demand-balancing effect of price reductions by offering deep discounts without the fear that these will backfire in low market states. Managerially, this observation points out that a firm employing threshold discounting should feature higher discounts compared to a firm using traditional price discrimination. To the extent that higher discounts boost word-of-mouth effects, our finding hints that the advantages of threshold discounting might be even higher once social effects are also accounted for.

13. Discussion

This paper studies the operational advantages of threshold discounting schemes when used by a capacity-constrained service provider that offers two vertically differentiated services to a random-sized population of strategically-acting customers. We show that threshold discounting outperforms traditional approaches on account of two phenomena: its responsive duality, which allows a firm to match its pricing and closing decisions to different market states, and a strategic scarcity effect, which improves the operational effectiveness of price reductions by signaling lower hot-period availability in high market states to strategic customers. Atypically, the presence of strategic customers increases firm profits in our context. We find that when offered through an intermediary, threshold discounting can lose its effectiveness if the deal specifications are chosen by the intermediary, due to incentive misalignment caused by commonly used contracts. We further expand the understanding of the design of threshold discounting schemes by showing that the optimal design involves a transparent threshold, early deal disclosure, as well as restricted discounting, and we suggest an idea for improving threshold discounting by providing focused incentives to specific consumer segments. Using real-world data, we estimate that threshold discounting schemes improve profit by up to 29% (6.1% on average) compared to traditional capacity management strategies.

Our model includes assumptions to avoid unnecessary complications to the analysis. We consider customer heterogeneity only with respect to valuation for the slow period, which is rich enough to both model customer preferences as heterogeneous and create vertical differentiation between the two service periods. In the classic threshold discounting studied in Section 10, the firm conditions both pricing and opening decisions during the slow period on the number of subscribers. A weaker form of threshold discounting is such that the number of subscribers merely affects pricing, and the firm is always open during both time periods. The results in this case are very similar, because when no discount is offered most customers shun the slow period; specifically, threshold discounting still grants the beneficial effects described in Section 10 and always outperforms price discrimination, but is less effective at managing fixed costs, so that when these are high enough seasonal closure becomes a better choice. In our analysis, we assume that all customers prefer the hot-period service to the slow-period service; that is, that service periods are vertically differentiated. This may not always be the case, as some consumers may have preferences that differ from the majority. Our results continue to hold under more general preference functions, where each period is preferred by a fraction of customers, except for the special case in which each period is preferred by exactly half of the consumer population, since in this case there is no need to rebalance demand through the use of discounts.

Our study is a first step towards understanding the operational benefit of threshold discounting offers, and it could be expanded in several interesting directions that, given their richness, we believe would deserve separate analysis. One would be to include competition, which we suspect will have interesting effects. Under competition, the ability of a given firm to redirect demand by closing down in certain time periods should be weakened, as some demand would spill over to competitors, while the ability to attract demand by reducing price should be strengthened, as some demand would be stolen from competitors. Another fruitful direction of investigation may involve looking at a multi-period choice model, which could open up the possibility for more sophisticated–and rewarding–demand manipulation schemes.

Part 3. Threshold Discounting Offers: Unintended Consequences and Incentive Conflicts

Threshold discounting offers–discounted price services that are valid only if at least a given number of customers show interest in them–have been pioneered by Groupon in the multi-billion-dollar online deals industry, and have been copied by hundreds of firms. We study these innovative discounting offers by modeling a capacity-constrained firm servicing a random-sized population of strategic customers in two representative time periods, a desirable hot period and a less desirable slow period. The use of a bi-variate customer preference distribution allows us to expand previous findings, to include situations in which discounts can effectively increase the market reached by the firm. We find that threshold discounting offers deliver the most value in situations in which, due to seasonal demand, capacity is scarce in certain periods and abundant in others; however, compared to traditional approaches (slow period discounting and closure) these offers deliver little value or can even be harmful in situations with chronically low demand, or when customers exhibit high frictional costs. When threshold discounts are offered through an intermediary, as often observed in practice, we find that the arrangements most used in practice distort the incentives of the intermediary, which can severely diminish the advantages of threshold discounting. Our results shed light on the possible reasons that may have led Groupon to unexpectedly discontinue threshold discounting offers. We conclude with a numerical study calibrated on data from the opera house Teatro Regio in Torino, Italy, where we consider a number of market and customer behavior scenarios to estimate that threshold discounting improves firm profits over traditional approaches by as much as 28% (7% on average).

14. Introduction

Groupon's value proposition since its foundation has been to help service providers attract customers during off-hours and better use their capacity. In order to lure customers into off-hours, Groupon has relied on deep discounts coupled with the use of an innovative discount structure, in which the discounted deals were valid only if a certain number of customers showed interest in the offering. The benefits of such deals, henceforth referred to as *threshold discounting* offers, have been celebrated in the business press as a way to leverage networking effects and economies of scale[31], and they have more recently received attention from the academic community as well. Unexpectedly, Groupon discontinued threshold discounting offers from its websites towards the end of 2012.[32] Its major competitors, one by one, followed its example in the years that followed.

The picture that emerges around threshold discounting offers is incomplete and controversial. Incomplete, because such offers have so far been studied mainly from a marketing perspective–ignoring for example capacity constraints–and the lack of an operational angle on threshold discounting is a serious matter, given that the daily deal industry has historically aimed at improving capacity utilization for hundreds of thousands of service providers around the world. [33]Controversial, because it is difficult to reconcile the celebrated advantages of threshold discounting offers with their progressive discontinuation on the part of the major players in the industry. Hence, two important questions remain open. Under which circumstances do threshold discounting offers provide value to a firm from an operational perspective? And why have these offers been discontinued by the major players?

Our work provides answers to both the above questions. Following Marinesi et al. (2013), we consider a capacity-constrained firm that offers its services to a random-sized population of strategically-acting customers who prefer to be served on a desirable "hot" time period over a less desirable "slow" time period (e.g., a movie theater on Saturday versus Monday evening). However, we broaden the scope of analysis by allowing customers to be heterogeneous in their valuation for both periods. This allows us to expand the results in Marinesi et al. (2013) by including the case in which some customers actually prefer the slow period over the hot period, and also by capturing the market-increase effect of discounting. This model allows us to provide compelling explanations for why

[31]See for example "Groupon's problem", Forbes, August 2012, http://goo.gl/AjG1x
[32]Groupon then CEO, Andrew Mason, resigned less than one year later.
[33]A notable exception is Marinesi et al. (2013), which constitutes a first important step in the study of threshold discounting offers from an operational perspective, however, more needs to be done to understand when threshold discounting offers provide the highest value and when should instead be avoided.

threshold discounting offers may have been discontinued. Indeed, we find that threshold discounting offers outperform traditional approaches when seasonal demand coupled with fixed capacity makes demand smoothing a priority for the firm. However, we also find that under certain conditions, threshold discounting provides little value, and can even reduce profit compared to traditional approaches. Specifically, we show that demand-starved firms, which probably constitute a big proportion of the service providers featured these days in daily deals websites, derive no operational benefit from a threshold discounting offer: this suggests that one reason why threshold discounting offers may have been discontinued could lie in a *lack of fit* between the advantages that they provide and the needs of those service firms that were attracted by daily deal websites. We also show that when threshold discounting is offered through an intermediary with high negotiating power (such as Groupon), the intermediary has strong incentives to prefer a much lower activation threshold and a lower price than what is in the interest of the service provider, and this may lead to substantial reductions in the profit of the firm; this result suggests that threshold discounting offers may have been discontinued due a second reason, i.e., due to *incentive misalignment* between the powerful intermediaries that dominate the industry and the service providers that need their services.

We use our expanded framework to test the surprising result in Marinesi et al. (2013) on the beneficial role of strategic customers. We find that some strategic customers are beneficial for a firm employing threshold discounting, , however, we also find that strategic customers have a concave effect on the firm profit, and therefore too many strategic customers in the population can potentially reduce the firm profit. To the extent that customers are becoming increasingly strategic, this finding may also constitute another possible reason for the discontinuation of threshold discounting offers.

We conclude with a numerical study calibrated on real data from the opera house Teatro Regio in Torino, Italy. We consider different market size distributions, customer preference distributions, and cost structures. In the over 200 scenarios examined, we find that threshold discounting increases firm profits by as much as 28% (average 7%) over the profit earned with the traditional approach.

Our work makes several contributions. Taking an operational perspective on threshold discounting, we provide prescriptions on when such offers should be used or avoided. We show that, in contrast with the strategic customer behavior literature in operations, strategic customer behavior is often beneficial, but it can be harmful due to its concave impact on firm profit. Our analysis also provides compelling explanations to the reasons that may have led Groupon and its competitors to discontinue threshold discounting offers, and cautions potential users of these discounts to the incentive conflicts inherent in the current modes of offering these discounts through intermediaries. We conclude our study with a quantification of the profit increase from offering threshold discounts (up to 28%) and we find that these benefits are highest in situations characterized by high market uncertainty.

15. Literature Review

Our work is related with the literature on quantity discounts, strategic customers, and demand manipulation via pricing, which can be found in Section 8. For the literature related to the role of intermediary in the supply chain, an excellent survey can be found in Belavina and Girotra (2012); our work departs from the existing literature on intermediaries by looking a the incentive misalignment created by threshold discounting offers, providing useful directional prediction on the terms of the deal when chosen by the intermediary, and relating it with the discontinuation of threshold discounting offers on the part of Groupon.

16. The Extended Model

In Section 9 we have studied a parsimonious model of threshold discounting, which captures two important effects of discounts: reducing margins and shifting demand across periods, as characterized in (9.4). However, there are situations in which discounts increase demand for a service, not just by cannibalizing sales from higher priced periods as in the base model, but also by reaching out to customers with lower valuations. Therefore, we now expand the model from Section 9 (henceforth referred to as the *base* model) by introducing a more sophisticated customer preference distribution, which allows us to capture this increased market effect of discounts. The role of this extended model is therefore twofold: testing the robustness of the findings from the base model and broadening their applicability.

16.1. **Preliminaries.** In this section, we expand the analysis from Section 9 by considering a population of customers that differ not only in their valuation for the slow period, but also in their valuation for the hot period–specifically, customer valuation vector (v_h, v_s) is drawn from the density function h, $h : [0, \bar{v}] \times [0, \bar{v}] \rightarrow \mathbb{R}_+$. The hot period is hereafter defined as the period preferred by the *majority* of customers, that is, the period in which more than half of the customers would make the highest surplus if the firm charged the same price on both periods. Formally, $\int_r^{\bar{v}} \int_0^{\tau_h} h(\tau_h, \tau_s)\, \mathrm{d}\tau_s \mathrm{d}\tau_h > \int_r^{\bar{v}} \int_0^{\tau_s} h(\tau_h, \tau_s)\, \mathrm{d}\tau_h \mathrm{d}\tau_s\, \forall r \in [\underline{v}, \bar{v})$. We also set $c = 0$ for simplicity.

Next, we briefly describe the equilibrium outcomes for the traditional approaches and for threshold discounting, and then compare their performance, putting emphasis on the differences with the results in the previous section. For the timeline of the approaches discussed, please refer to the Figures displayed in chapter two.

16.2. **The Traditional Approaches.** Under closure, customers visit the firm as long as their valuation for the hot period is higher than the price charged by the firm, r_h. The profit of the firm is then

$$\Pi_c = \int_0^{+\infty} \left[r_h \min(k, \alpha_h^c x) - c_F \right] \mathrm{d}G(x),$$

where $\alpha_h^c = \int_{r_h}^{\bar{v}} \int_0^{\bar{v}} h(\tau_h, \tau_s)\, \mathrm{d}\tau_s \mathrm{d}\tau_h$ is the fraction of the market with valuation for the hot period higher than r_h. Figure 16.1a illustrates customer visit decision as a function of the valuation vector (v_h, v_s).

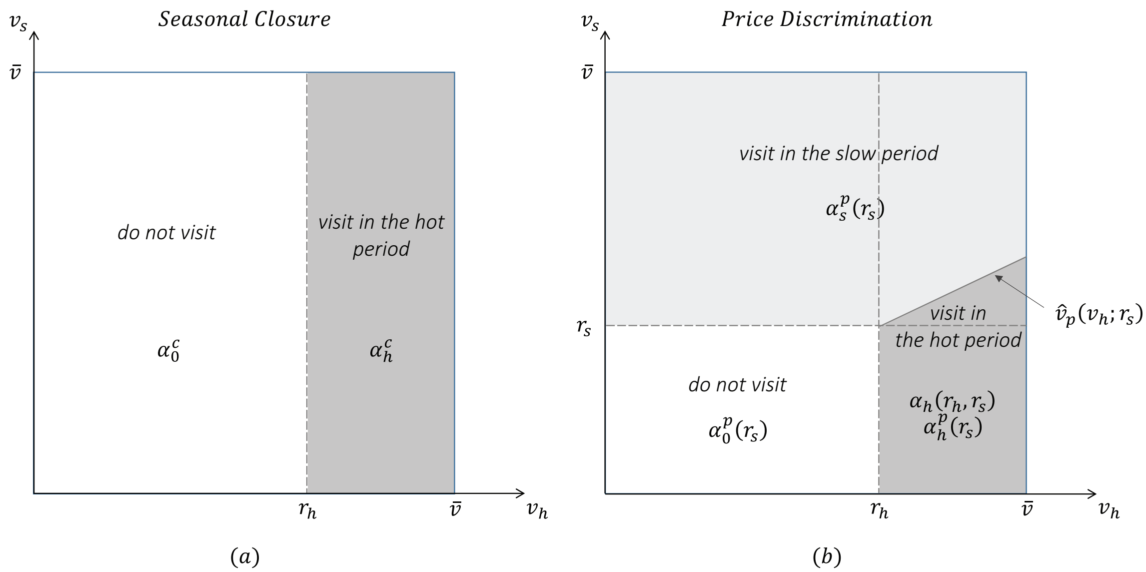


Figure 16.1: Customer equilibrium visit strategies as a function of their valuation vector under Seasonal Closure (a) and Price Discrimination (b)
The plane displayed in the figure is the support of the general bi-variate customer preference distribution h.

Under price discrimination (Figure 16.1b) when the price vector (r_h, r_s) is charged, the customer's decision depends on which of four valuation clusters they belong to: customers whose valuation is lower than the price for both periods do not visit the firm, customers whose valuation allow for a positive surplus only in one period visit during that period, and customers who would make a positive surplus in both periods visit in the slow period iff their slow period valuation v_s is sufficiently higher than their hot period valuation v_h, i.e., higher than $\hat{v}_p(v_h; r_s)$, obtained as the solution to

$$(\hat{v}_p(v_h; r_s) - r_h) \int_0^{+\infty} \min\left(1, \frac{k}{\alpha_h^p(r_s)\, x}\right) \mathrm{d}G_c(x) = (\hat{v}_p(v_h; r_s) - r_s) \int_0^{+\infty} \min\left(1, \frac{k}{\alpha_s^p(r_s)\, x}\right) \mathrm{d}G_c(x),$$

where $\alpha_h^p(r_s) = \int_{r_h}^{\bar{v}} \int_0^{r_s + (\hat{v}_p(\tau_h; r_s) - r_s)(\tau_h - r_h)(\bar{v} - r_h)^{-1}} h(\tau_h, \tau_s)\, \mathrm{d}\tau_s \mathrm{d}\tau_h$ represents the fraction of customers visiting in the hot period, and $\alpha_s^p(r_s) = \int_0^{\bar{v}} \int_{r_s + \left((\hat{v}_p(\tau_h; r_s) - r_s)(\tau_h - r_h)(\bar{v} - r_h)^{-1}\right)^+}^{\bar{v}} h(\tau_h, \tau_s)\, \mathrm{d}\tau_s \mathrm{d}\tau_h$ represents the fraction of customers visiting in the slow period.

The four valuation clusters can be rearranged so as to form three groups: hot period visitors (α_h^p), slow period visitors (α_s^p), and non-visitors (α_0^p). The fact that not all customers visit the firm is an important difference with our analysis in Section 9, which has implications for the firm's pricing decision, as we discuss next.

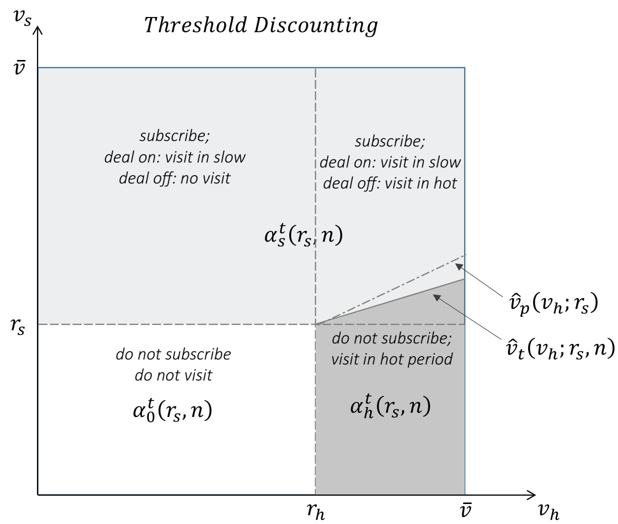


FIGURE 16.2: Customer equilibrium subscription and visit strategies as a function of their valuation vector under Threshold Discounting *The plane displayed in the figure is the support of the general bi-variate customer preference distribution h.*

The effect of a discount θ on the profit of a firm employing price discrimination can be meaningfully grouped into three terms:

$$\frac{d}{d\theta}\Pi_p(\theta) = \underbrace{\int_0^{+\infty} \frac{d\alpha_h^p(\theta)}{d\theta} r_h \left(1_{x<k\left(\alpha_h^p(\theta)\right)^{-1}} - 1_{x<k\left(\alpha_s^p(\theta)\right)^{-1}}(1-\theta)\right) x\mathrm{d}G(x) +}_{\Pi'_{p-op}(\theta),\, operational\,(\pm)}$$

$$+\left[\underbrace{-\int_0^{+\infty} \min\left(k, \alpha_s^p(\theta)\, x\right) \mathrm{d}G(x)}_{\Pi'_{p-mg}(\theta),\, margin\,(-)} \underbrace{-\int_0^{k\alpha_s^p(\theta)^{-1}} \left(r_h(1-\theta)\right) \frac{d\alpha_0^p(\theta)}{d\theta} x\mathrm{d}G(x)}_{\Pi'_{p-im}(\theta),\, increased\, market\,(+)}\right], \tag{16.1}$$

where $\alpha_i^p(\theta), i \in \{h, s, 0\}$ is short notation for $\alpha_i^p(r_s(\theta))$.

In line with the analysis of the base model, a higher discount in the slow period shifts customers from the hot to the slow period, which is captured in the first component, $\Pi'_{p-op}(\theta)$;[34] it also reduces the margin earned in that period, which is captured in the second component, $\Pi'_{p-mg}(\theta)$. Unlike our analysis in Section 9, however, an increase in the discount offered in the slow period also increases the fraction of the market that can make a positive surplus in the slow period, α_0, thereby increasing

[34]We henceforth assume that the service periods are substitute goods, so that a discount in the slow period attracts customers from the hot period i.e., in this case $\frac{d\alpha_h^p(\theta)}{d\theta} > 0$.

sales in that period–captured in the third component, $\Pi'_{p-im}(\theta)$. To summarize, a discount induces a demand shift towards the slow period (first component) which works in addition to the classic lower margin-higher sales tradeoff (last two components). Unlike our analysis in Section 9, the hot period is not always the period with the most visitors: in some cases, the firm may be better off discounting the slow period so much that the increase in sales makes it busier than the hot period: at that discount level, the operational effect $\Pi'_{p-op}(\theta)$ negatively affects profit because it makes demand across periods more unevenly distributed, thus increasing the mismatch between supply and demand–yet the increase in slow period sales more than compensate for it.

The profit for a firm employing price discrimination is given by

$$\Pi_p = \int_0^{+\infty} \left[r_h \min\left(k, \alpha_h^p\left(r_s^p\right)x\right) + r_s^p \min\left(k, \alpha_s^p\left(r_s^p\right)x\right) - 2c_F\right] \mathrm{d}G\left(x\right), \tag{16.2}$$

where $r_s^p = \arg\max_{r_s} \Pi_p\left(r_s\right)$ subject to $r_s \leq r_h$.

16.3. **Threshold Discounting.** Figure 16.2 illustrates customer subscription and visit strategies as a function of the valuation vector (v_h, v_s). Similarly to price discrimination, customers can be grouped into four valuation clusters. Customers in the first cluster are those who make a positive surplus neither during the hot period nor during the discounted slow period, so they do not subscribe and do not visit (α_0^t); customers in the second cluster are those who make a positive surplus only in the discounted slow period, and they subscribe to the deal, visit during the slow period if the deal is active, and they do not visit otherwise; customers in the third cluster are those who can make a positive surplus only in the hot period, and they do not subscribe and visit in the hot period regardless of the outcome of the deal; and finally, customers who can make a positive surplus both in the hot and in the discounted slow period act to maximize their expected surplus. Specifically, if their slow period valuation is sufficiently higher than their hot period valuation, i.e., higher than $\hat{v}_t\left(v_h; r_s, n\right)$, they act as customers in the second cluster, otherwise they act as customers in the third cluster, with $\hat{v}_t\left(v_h; r_s, n\right)$ being the slow period valuation that makes a customer with valuation v_h indifferent between the two options, which is the solution to:

$$(v_h - r_h) \int_{n/\alpha_s^t(r_s,n)}^{+\infty} \min\left(1, \frac{k}{\alpha_h^t(r_s,n)\,x}\right) \mathrm{d}G_c(x) = \left(\hat{v}_t(v_h; r_s, n) - r_s\right) \int_{n/\alpha_s^t(r_s,n)}^{+\infty} \min\left(1, \frac{k}{\alpha_s^t(r_s,n)\,x}\right) \mathrm{d}G_c\left(x\right),$$

where $\alpha_h^p(r_s, n)\,\tau_h$ represents the fraction of customers who visit during the hot period when the deal is active, and $\alpha_s^p(r_s^p, n)$ represents the fraction of customers who, conditional on having subscribed, visit during the slow period when the deal is active, and are defined in Section C.1. The strategic scarcity effect studied in Section 10.2 continues to hold in the current model, that is, conditional on the deal being on, customers learn that the market size is higher than anticipated, leading to a larger fraction of customers visiting during the slower period under threshold discounting compared to price discrimination when the same discount is offered (Section C.6, Lemma 10). Graphically, this is reflected in the fact that the slope of the border between α_h^t and α_s^t is more distant from the 45 degree line compared to the border between α_h^p and α_s^p when the same discount is offered.

The effect of a higher discount on the profit of threshold discounting can be decoupled into an operational effect, a margin effect, and an increased market effect similarly to what is done in (16.1) for the case of price discrimination, plus a threshold effect, and can be found in C.2.

The profit for a firm employing threshold discounting is

(16.3)

$$\Pi_t = \int_0^{\frac{n^t}{\alpha_s^t(r_s^t,n^t)}} \left[r_h \min(k, \alpha_h^c x) - c_F\right] \mathrm{d}G(x) + \int_{\frac{n^t}{\alpha_s^t(r_s^t,n^t)}}^{+\infty} \left[r_h \min\left(k, \alpha_h^t\left(r_s^t, n^t\right) x\right) + r_s^t \min\left(k, \alpha_s^t\left(r_s^t, n^t\right) x\right) - 2c_F\right] \mathrm{d}G(x)\,,$$

where $\left(r_s^t, n^t\right) = \max_{r_s,n} \Pi_t\left(r_s, n\right)$ subject to $r_s < r_h$, $n > 0$ is the profit maximizing decision under threshold discounting. For the details of the equilibrium analysis, see Section C.1.

16.4. **Comparison of Threshold Discounting with the Traditional Approaches.**

Theorem 14. *Consider a market size distribution G with support over $\mathbb{R}_+$. Then threshold discounting outperforms closure, i.e. $\Pi_t > \Pi_c$.*

The comparison between threshold discounting and closure confirms the result obtained in the base model, because the driver of the advantage of threshold discounting are still in place: as in our previous analysis, threshold discounting can exploit the variability in the market to its advantage, expanding capacity and smoothing demand when the market size realization is high enough, and saving on fixed costs and avoiding sales cannibalization when the market size is low. The additional sales accrued via the discount in the slow period often times augment the threshold discounting advantage by allowing the firm to more easily break even in the slow period.

On the other hand, the comparison between threshold discounting and price discrimination is more elaborate once the increased market size effect of a discount is taken into account.

Theorem 15. *Consider a market size distribution G with support over $\mathbb{R}_+$. Then threshold discounting outperforms price discrimination, that is, $\Pi_t > \Pi_p$, if*

$$\Pi'_{p-op}(\theta_p) > 0, \tag{16.4}$$

and either $c_F > 0$ or $\alpha_h^c - \alpha_h^p(\theta_p) > (1-\theta_p)\,\alpha_s^p(\theta_p)$, where θ_p is defined as $\theta_p = \arg\max_\theta \Pi_p(r_s(\theta))$ subject to $\theta \geq 0$.

The above theorem provides sufficient conditions for threshold discounting to outperform price discrimination. The second condition requires that fixed costs are positive, or else that under price discrimination the profit lost due to sales cannibalization offsets the profit gained thanks to the increased market size effect. This ensures that neither of the traditional approaches dominates the other under all market size realizations. This is arguably very likely to hold, hence the result of the theorem bears predominantly on condition (16.4): this condition is insightful, and states that under price discrimination, in equilibrium, the operational (marginal) effect of a discount defined in (16.1) is positive. This condition is important because it relates to the decision of a firm employing price discrimination, thereby providing information on the conditions under which the firm operates. When this condition holds, the discount set by the firm under price discrimination has a positive operational impact on profit, formally $\int_0^{\theta_p} \Pi'_{p-op}(\theta)\,\mathrm{d}\theta > 0$, which means that the firm benefits from the demand-shift induced by the discount (Section C.6, Lemma 11); furthermore, this condition implies that the hot period is busier than the slow period and subject to capacity shortages (Section C.6, Lemma 12); finally, it also implies that the sum of the margin and increased market (marginal) effects on profit is negative, which means that the firm pushes the discount up to a level that sacrifices on margin/sales in order to gain on the operational side. In short, condition (16.4) characterizes those situations in which, due to seasonality of demand and capacity shortages, demand smoothing is a primary driver both for the firm's pricing decision and for its profitability. Under these conditions, threshold discounting outperforms price discrimination.

Theorem 15 suggests that threshold discounting is no longer guaranteed to outperform price discrimination. This happens because the extended model also captures situations in which discounts are used primarily as a way to reach out to low valuation customers, and in such situations, the operational advantages of threshold discounting may lose their edge. It is therefore of interest to isolate situations in which threshold discounting delivers no value to the firm, or may even be harmful. To this aim, we now consider the case of a bounded support for the market size distribution G, and consequently restrict the firm employing threshold discounting to pick a meaningful threshold, i.e., a threshold such that the deal is active and inactive with some positive probability.

Theorem 16. *Consider a market size distribution G with open bounded support $(\underline{x}, \bar{x}) \subset \mathbb{R}_+$; then*

- *Price discrimination strictly outperforms threshold discounting if $\bar{x} \leq k$ and $c_F < \bar{c}_F$, that is, $\Pi_p > \Pi_t$; and*
- *Closure strictly outperforms threshold discounting if $\bar{x} \leq k$ and $c_F > \breve{c}_F$, that is, $\Pi_c > \Pi_t$,*

where $\bar{c}_F = r_h \left(\alpha_h^p \left(r_s^p\right) - \alpha_h^c\right) \underline{x} + r_s^p \alpha_s^p \left(r_s^p\right) \underline{x} > 0$ and $\breve{c}_F = r_h \left(\alpha_h^p \left(r_s^p\right) - \alpha_h^c\right) \bar{x} + r_s^p \alpha_s^p \left(r_s^p\right) \bar{x}$, and r_s^p is defined as in (16.2).

The above theorem provides conditions for each of the traditional approaches (price discrimination and closure) to outperform threshold discounting. When a firm is *demand-starved*, that is, when the firm is always going to have spare capacity in both service periods, the firm has no need for demand smoothing, hence the strategic scarcity effect provides no value. When, in addition, fixed costs are low (high) enough, opening during both periods and setting the appropriate price is always better (worse) than closing, hence the responsive duality advantage also no longer provides value. More specifically, when fixed costs are low enough, the firm's optimal decision is simply to offer the same discount, one that optimally trades off higher margins and higher sales, independently from the market size, since no capacity shortages occur. Vice versa, when fixed costs are high enough, the firm optimal decision is to close down, independently–again–from the market size. In these cases, simple approaches such as price discrimination and closure provide–respectively–the firm with just what is needed, while any market-responsive approach that implements different pricing/opening decisions in different market outcomes, as threshold discounting needs to do in order to ensure customer participation, is going to reduce the firm profit. It follows that in these cases threshold discounting performs worse than the most appropriate between the traditional approaches.

Note that we are assuming that a firm running a threshold discounting offer does not incur any additional costs. However, threshold discounting arguably requires the firm to undertake costly activities and investments, such as creating and maintaining a website, paying for server space, employing sales staff, IT support, etc. Hence, for threshold discounting to be a profitable alternative to traditional approaches, the profit gap relative to the traditional approaches must not only be just positive, but also cover at least the extra costs necessary to run a threshold discounting offer.

The above results show that threshold discounting is not a one-size-fits-all approach, and in some cases a discounted deal without any threshold or simply closing the firm in slow periods can be a better choice, specifically when a firm is demand starved and has low or high costs of operations, respectively. Conditions in Theorem 16 are likely to hold for young businesses and struggling businesses alike. These categories probably represent a large portion of the firms featured on daily deals websites. Hence, one reason for why threshold discounting offers have been discontinued by many players in the industry could have been the lack of fit between the (operational) benefit associated with threshold discounting offers and the need of those (demand-starved) firms that seek to be featured on daily deal websites. Nonetheless, whether discontinuing threshold discounting was a savvy long-term decision remains questionable, as long-term value is less likely to come from demand-starved businesses, and is more likely to come from healthier businesses with conspicuous seasonal demand–exactly the type of businesses that would benefit the most from threshold discounting offers.

Taken together, Theorems 15 and 16 expand the results from our analysis in Section 9 by accounting for the increased market effect of discounts and for situations characterized by low market prospects. Our new results confirm that responsive duality and strategic scarcity are the drivers of the superiority of threshold discounting offers; in particular, these two advantages make threshold discounting superior to seasonal closure when the market size exhibits enough variability, and make it superior also to price discrimination in settings where, in addition, discounts are primarily used as a way to smooth demand. On the other hand, we have also shown that threshold discounting often fails to deliver value and can even hurt the firm profits for demand-starved firms.

17. Additional Analysis

17.1. **The Impact of Strategic Customer Behavior on the Firm Profit.** In this section, we extend our analysis to consider a mixed population in which a fraction γ of customers are strategic, and the remaining fraction $1-\gamma$ are non-strategic, in that they do not account for the strategy of other customers. Consistent with Marinesi et al. (2013), a non-strategic customer simply compares the hot period surplus $v_h - r_h$ with the slow period surplus $v_s - r_s$, and visits in period that yields the higher surplus if positive, or else does not visit; if she plans on visiting in the slow period, she also subscribes to the deal.

In our discussion following theorem 15 we observed that when discounts can reach new portions of the market, additional demand smoothing is not necessarily beneficial to the firm. In such situations, therefore, strategic customers could be harmful to the firm, due to their augmented strategic response to slow period discounts upon learning that the deal is active. The profit expression for the firm is similar to (10.3) and therefore has been relegated to the Appendix (C.3).

Theorem 17. *Assume that* $\gamma = 0$*, and also that*

(1) $\Pi_p(\theta)$ *and* $\Pi_t(\theta, n)$ *are unimodal in* $\theta, \forall n$*;*
(2) $\Pi'_{p-mg}(\theta) + \Pi'_{p-im}(\theta)$ *is unimodal in* θ*; and*
(3) $\Pi'_{p-op}(\theta_p) > 0$,

where $\theta_p = \max_\theta \Pi_p(r_s(\theta))$*. Then*

$$\frac{d}{d\gamma}\Pi_t|_{\gamma=0} > 0.$$

When the conditions in the above theorem hold, the profit of a firm employing threshold discounting with a population of non-strategic customers increases if at least some of the customers are strategic. In other words, under these conditions, having some strategic customers in the population leads to higher profit compared to having none.

The first two conditions in the theorem are regularity conditions. The first states that the optimal discount level under price discrimination and threshold discounting is given by the first order condition. The second requires the sum of the margin and the increased market effects under price discrimination to be unimodal in the discount level θ. The third condition is the same condition used in theorem 15, and characterizes settings in which demand smoothing is a priority for the firm, and threshold discounting outperforms both traditional approaches. Hence, those in which threshold discounting is best placed to operate, i.e., in which (16.4) holds, are often the same conditions in which (some) strategic customers are beneficial to the firm. It is possible that a high fraction of strategic customers may increase or reduce the firm profit because, as commented earlier, an excessive level of demand smoothing may hurt profits (in particular, one can show that demand smoothing has a concave impact on profit at any price point, see Section C.12, Lemma 15). This issue is investigated in the numerical study (Section 18). Overall, Theorem 17 complements our result in Section 10.3, reinforcing our finding on the surprisingly beneficial nature of strategic customers for a firm employing threshold discounting.

One may wonder how strong the above regularity conditions are. The answer is: not too strong, because increasing the discount charged by the firm tends to have a concave effect on each of the three components in (16.1). More specifically, the increased market effect of discount on profit for instance tends to be concave because increasing sales has progressively less value as the discount increases; similarly, the operational effect of discount on profit (condition 1, second part) is always quasi-concave (Section C.6, Lemma 11) and tends to be concave because, as commented earlier, the impact of demand smoothing on profit is concave. The margin effect of discount on profit is always concave, since for higher discounts the margin is lost on a higher amount of sales. For a formal analysis on when these conditions hold, see Lemma 13 in the Appendix.

17.2. **Mediated Threshold Discounting.** In the most popular implementations of threshold discounting, the service provider offers threshold discounts through an intermediary (such as Groupon), which features the deal on its website in exchange for a commission. The main advantage of going through a third party is probably to reach a larger number of customers: in this case, threshold discounting can generate word-of-mouth effects, as Jing and Xie (2011) analyze. From the operational perspective of our study, however, an intermediary provides no clear advantage to the firm, though the need for an intermediary may still arise as a way for a firm to obtain the necessary visibility and reach customers, possibly because customers are not aware of the firm's business. If threshold discounting is offered through an intermediary, decision rights are a key consideration: who decides on the characteristics of the deal (the activation threshold and the discounted price), the service provider or the intermediary? If it is the service provider, then the intermediary is simply an extra cost, and threshold discounting is preferable to traditional approaches only insofar as the advantages outweigh the intermediation costs. In this case our analysis above applies with the cost of intermediation subtracted from the firm's profit.

In practice, however, the intermediary has a large role in shaping the characteristics of the deal, because of the inexperience of the service provider, for example, or because of the intermediary's high bargaining power. In these cases, it is imperative to learn how the incentives of the intermediary differ from the those of the firm. Based on our interactions with management of daily deals businesses, the contract arrangement most often used in practice, possibly due to its simplicity, observability and objectivity, is such that the intermediary earns a percentage of the revenues from those customers that subscribed to the offer through its website. In this case, the profit for the service provider (subscript t-med) and the intermediary (subscript IN) for any deal (r_s, n) and positive intermediation fee η are then given by

$$\Pi_{t-med}(r_s, n|\eta) = \int_{n/\alpha_s^t(r_s,n)}^{+\infty} \left[r_h \min\left(k, \alpha_h^t\left(r_s, n\right) x\right) + (1-\eta)\, r_s \min\left(k, \alpha_s^t\left(r_s, n\right) x\right) - 2c_F\right] \mathrm{d}G\left(x\right) + \\ + \int_0^{n/\alpha_s^t(r_s,n)} \left[r_h \min(k, \alpha_h^c x) - c_F\right] \mathrm{d}G\left(x\right)$$

$$\Pi_{IN}\left(r_s, n|\eta\right) \triangleq \eta \int_{n/\alpha_s^t(r_s,n)}^{+\infty} r_s \min\left(k, \alpha_s^t\left(r_s, n\right) x\right) \mathrm{d}G\left(x\right).$$

Theorem 18. *Under mediated threshold discounting*

- *the intermediary chooses a lower slow period price than the firm would, for any given activation threshold n, i.e. $r_s^{IN}\left(n\right) < r_s^t\left(n\right)\ \forall n > 0$;*
- *the intermediary chooses a lower activation threshold than the firm would, for any given slow period price r, if $\gamma \leq \bar{\gamma}\left(r_s\right)$, i.e. $\forall r_s \exists \bar{\gamma}\left(r_s\right) \in (0, 1] : \gamma \leq \bar{\gamma}\left(r_s\right) \Rightarrow n_s^{IN}(r_s) < n_t\left(r_s\right)$; and*
- *the firm earns a lower profit, even when $\eta \to 0^+$,*

where $r_s^{IN}(n) = \arg\max_{r_s} \Pi_{IN}(r_s, n|\eta)$ *s.t.* $r_s \leq r_h$, $n_s^{IN}(r_s) = \arg\max_n \Pi_{IN}(r_s, n|\eta)$ *s.t.* $n > 0$, $r_s^t(n) = \arg\max_{r_s} \Pi_t(r_s, n)$ *s.t.* $r_s \leq r_h$, *and* $n_t(r_s) = \arg\max_n \Pi_t(r_s, n)$ *s.t.* $n > 0$.

The profit of the intermediary differs from the profit of the service provider in three important ways. First, the intermediary earns profit only on customers who purchase during the slow period; second, the intermediary does not incur any additional fixed cost if the service provider opens also in the slow period; and third, the intermediary earns profit only when the deal is active. The first two differences provide strong incentives for the intermediary to charge a *lower slow-period price* than the service provider would. One reason is that the intermediary has much higher incentives to shift demand to the slow period–for he earns nothing when customers purchase on the hot day–and this is best achieved by lowering the price. Another reason is that the intermediary is willing to open during the slow period as long as this brings one cent more in revenues, while the service provider is wary of the fixed costs that such decision brings along.

The second and third differences imply instead that, compared to the service provider, the intermediary prefers a deal that is much more likely to be active, meaning a *lower activation threshold*. The reason is that the intermediary takes all the benefits of an active deal–higher revenues during the slow period–without getting most of the costs associated with it–costs of opening, since these are incurred by the service provider, and cost due to the cannibalization of the hot period sales by the slow period, since the intermediary gains nothing from selling during the hot period. The only cost for the intermediary in lowering the activation threshold comes from reducing the strategic scarcity effect–a lower threshold sends a weaker signal to strategic customers upon deal activation–which results in lower sales in the slow period.[35] However, this cost is often negligible. Too see why, one must consider the interaction between the two effects in the Theorem 18: once the intermediary lowers the price during the slow period, demand will further shift to the hot period; this demand shift weakens the strategic scarcity effect, which is based on the difference in availability between the two periods, and in how signaling a high market size via the deal activation makes such a difference more prominent in the eyes of the customers. Once the additional price reduction favored by the intermediary has weakened the strategic scarcity effect, lowering the threshold is going to have little consequences on the slow period sales.

In summary, the intermediary is in many cases better off with a lower activation threshold and a lower price, both of which undermine the advantages of threshold discounting for the service provider. In this case responsive duality is severely diminished, both because the deal would be activated in market states in which it would be best not to activate the deal, and because an excessive fraction of demand would be redirected to the slow period, reducing the operational benefit of price discriminating between periods. Further, the strategic scarcity effect would also be less than optimal on account of the lower threshold. Our numerical study strongly supports both of the above predictions, with the intermediary choosing a lower price and a much lower threshold (often almost equal to zero) than the firm would (see Section 18). This logic indicates that the deal preferred by the intermediary, one with a very low–if not zero–threshold and a deep discount,

[35] When the hot period is busier than the slow period; otherwise, reducing the strategic effect increases sales in the slow period, and the intermediary always prefers a lower threshold than the service provider does.

could substantially reduce the profit of the service provider due to incentives misalignment, that is, over and above the fee charged by the intermediary. Our observations of numerous Groupon-like websites suggest that the characteristics of many online discounted deals appear consistent with the above logic. We therefore conjecture that threshold discounts work well when administered directly by the service provider, but not when administered by an intermediary.

17.3. **Transaction Cost of Subscription.** Our analysis of threshold discounting so far assumes that customers incur a negligible (positive but arbitrarily small) effort cost when subscribing to a threshold discounting offer. Suppose instead that customers incur a non-negligible transaction cost ϕ to subscribe to a threshold discounting offer (see Section C.9 for the equilibrium conditions). How would this change the performance of such offers?

Theorem 19. *When customers incur a non-negligible transaction cost ϕ to subscribe to a threshold discounting offer, in comparison to a case with negligible subscription costs*

- *Total sales are lower at any price point, i.e.,* $\alpha_h^t(r,n)+\alpha_s^t(r,n) > \alpha_h^t(r,n;\phi,\eta)+\alpha_s^t(r,n;\phi,\eta)$ $\forall r,n,\phi,\eta > 0$,
- *A larger fraction of visiting customers is served during the hot period relative to the slow period at any price point, i.e.* $\alpha_h^t(r,n)\left(\alpha_s^t(r,n)\right)^{-1} < \alpha_h^t(r,n;\phi,\eta)\left(\alpha_s^t(r,n;\phi,\eta)\right)^{-1}$ $\forall r,n,\phi,\eta > 0$, *and*
- *Profit is lower;*

as $\phi \to 0^+$, threshold discounting with negligible and with non-negligible transaction cost become outcome equivalent, that is, result in the same sales and profit in each period for every market size realization.

When customers incur a transaction cost to subscribe to a threshold discounting offer, fewer customers will subscribe at any given price point, due to the additional transactional cost that they will have to incur; as a consequence, fewer customers will be able to obtain the discount and therefore visit the firm during the slow period. As a result, total sales are lower because a higher fraction of the market is unable to access the firm services. In terms of expected surplus, the existence of a transaction cost weighs on customers as an increase in price would, therefore shifting demand towards the hot period. In short, the existence of transaction costs lowers sales and reduces the ability of threshold discounting to shift demand towards the slow period. As a result, profit for the firm is also reduced. As transaction costs become smaller, this model converges to the model with negligible transaction costs studied in Section 16.3. To the extent that customer transaction costs are becoming lower and lower, thanks to widespread diffusion of smartphones and one-touch apps, the assumption of negligible subscription cost is a good approximation, and threshold discounting offers are going to provide even more value in the future.

18. Numerical Study

In this section, we present the results of a numerical study that helps us illustrate the advantages of threshold discounting and complement our theoretical results. We consider the usage of threshold discounting at a potential service provider, the opera house Teatro Regio located in Torino, Italy. We extrapolate cost data from their 2013 balance sheet, and we use their pricing data to guide our choice for customers' inter-temporal preference parameters. Table 18.1 illustrates the values chosen for each parameter and the criteria employed. In the absence of complete data on customer preferences, we use the simplest distribution that results in a seasonal demand pattern while keeping computation simple–a triangular distribution–and we model both a strategic and a non-strategic population of customers. As per the cost structure, we consider alternate choices to study its effect on the performance measures of interest. Specifically, we consider nine different market size distributions, eight different preference distributions, and three different cost structures–the actual cost structure of Teatro Regio, plus two additional cases–and we simulate all possible combinations of these parameters, for a total of 216 scenarios examined.

Figure 18.1 (a) shows the profit gains of threshold discounting over the traditional approaches (i.e., the best between closure and price discrimination) for each of the 216 scenarios simulated. In 16% of the scenarios, threshold discounting performs worse than the traditional approaches, but the difference in performance is small, with an average loss of 0.2%, up to a maximum loss of 0.4%. In the 84% remaining scenarios, threshold discounting leads to profit gains that are often substantial, up to about 28% (average 7%). The difference in performance across the 216 scenarios can be partly explained using the dilemma score, defined as $-|\Pi_p - \Pi_c|\,(\Pi_p + \Pi_c)^{-1}$. This score has a correlation of 0.62 with profit gains, and it roughly captures the service provider's dilemma of having to choose between price discrimination and closure in the face of market size uncertainty: the closest are the profit of the two approaches in expectation, the higher is the dilemma score, and the higher is (ceteris paribus) also the ex-post profit foregone by choosing the least performing approach, thanks to the single crossing property of the ex-post profit function proven in Lemma 2 (that this property holds also in the extended model it is easy to show). In essence, the dilemma score captures the responsive duality advantage of threshold discounting, which allows the firm to indirectly respond to the unobserved market size. Figure 18.1 (b) compares profit gains with and without strategic customers: strategic customers are in 89.8% of the cases beneficial for a firm employing threshold discounting, improving profit by a modest 0.4% on average, up to 4.3%; this may seem like a small improvement, but it comes as a free lunch, without any further cost or intervention on the part of the firm. In the remaining 10.2% of cases, the impact of strategic customers is negative, but hardly noticeable.

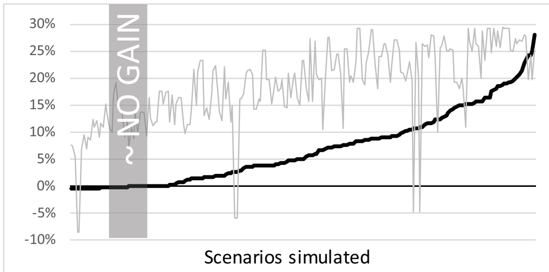
Parameter	***Value(s) Considered***	***Source***
Capacity, k	1500	The actual capacity of Teatro Regio is 1582 seats, of which 1530 are proper seats (the rest being stools). We rounded down to 1500
Market size, $\tilde{x}$	Uniform distribution: mean 3000, 4000, and 5000; and widths 2000, 4000, and 6000, for a total of 9 distributions	Given the popularity of the Teatro Regio in the last years, we consider the average potential market size to be between 2 and 3.33 times the single-period capacity. We have no information on demand variability, thus we use different mean-preserving spreads to study the impact of market uncertainty on performance metrics of interest.
Fixed cost, c_F	k€ 30, 50, 70	The fixed costs that could be saved by closing down on a given night at Teatro Regio are estimated to be about 50K€, which comprises the per-show payroll for external performers and the cost of utilities. We also consider higher and lower values in order to capture a broad range of situations and study the impact of fixed costs on the performance metrics of interest.
Full price, r_h	€ 130	The price charged for prime-time performances during the season (if we exclude the day of inauguration).
Upper valuation, $\bar{v}$	€ 180, 210	We examine two potential upper-bound valuations.
Lower valuation, $\underline{v}$	€ 0, 40	We examine two cases, € 40 the lowest price charged at Teatro Regio for "slow" periods, € 0 the extreme case such that a customers obtains no value from attending the event.
Customer valuation density function, h	$h\left(v_h, v_s\right)$equal to: $\left(\bar{v} - \underline{v}\right)^2/2$ if $v_h \geq v_s$ 0 otherwise	We consider the case of a triangular distribution, since this provides a simple closed form to compute the fraction of visitors in each period as a function of the threshold valuation $\hat{v}$ defined in (**??**).
Unit cost, c	€ 0	The marginal cost of issuing a ticket is negligible.
Fraction of strategic customers, γ	0, 1	We consider the two extreme cases of a strategic population and of a non-strategic population
Intermediation fee, η	50%	The usual fee charged by intermediaries in the industry (see for example http://goo.gl/J4yNuH)

TABLE 18.1: Parameter values employed in the numerical study

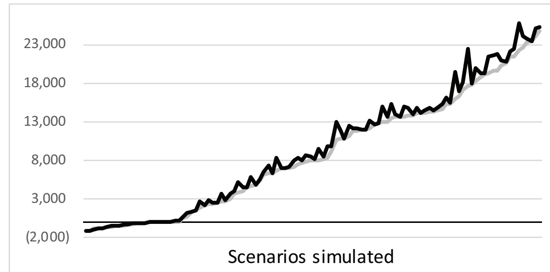
Figure 18.1 (c) shows how profit gains change as market uncertainty increases while the mean market size remains constant, for different levels of fixed costs. Interestingly, profit gains are higher when market uncertainty increases, suggesting that threshold discounting is particularly useful in the presence of high market uncertainty. A finer search reveals that profit gains actually increase in market uncertainty in all 216 scenarios considered. The fact that profit gains increase in the fixed costs is not a general property, and is due to the fact that price discrimination earns higher profits than closure in more than 60% of the scenarios simulated, hence higher fixed costs within our pool of scenarios boost the responsive duality advantage by improving the service provider dilemma score, thus increasing profit gains. Figure 18.1 (d) shows the impact of market uncertainty on the profit of threshold discounting, price discrimination, and closure for a representative set of parameters. Note that higher uncertainty reduces the profit of both closure and price discrimination, as one would expect given the concavity of profit with respect to market size realization. However, higher market uncertainty is much less of a threat for a firm using threshold discounting, as it reduces profit to a lower extent or, contrary to intuition, it may even increase profit, as in the left part of (d). This is because higher market uncertainty amplifies the responsive duality advantage of threshold discounting, and this effect may offset the negative effects traditionally associated with market uncertainty.

Figure 18.2 plots the equilibrium slow period price and the activation threshold under mediated threshold discounting when the intermediary chooses the terms of the deal, and compares the intermediary's preferred deal (black) with the deal that maximizes the profit of the firm (dark gray). In all the 216 scenarios simulated, the intermediary chooses a much lower threshold compared to what would be optimal for the firm, resulting in the deal being active 99% of the times or more. This provides strong support for why daily deal websites may have decided to discontinue threshold discounting. In all the scenarios, the intermediary also chooses a lower price than what would be optimal for the firm (8% lower on average). Both for the activation threshold and the price, the supply chain optimal decision (pale grey) almost consistently stands between the individualistic choice of the firm and of the intermediary, as evidence that their incentives are not only different, but also divergent with respect to the common optimal.

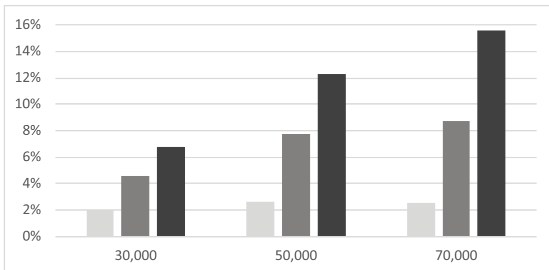
Finally, one may wonder whether threshold discounting could improve profits compared to more sophisticated capacity management techniques, such as advance selling. Since the comparison proves intractable analytically, we tested it using our pool of scenarios. We find that threshold discounting



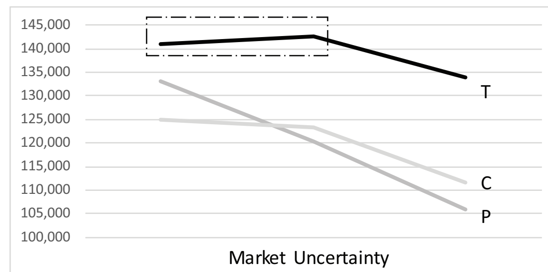
(a) Expected Profit Gains in our cases
(black=profit gains, grey dotted=dilemma score)
ordered by increasing percentage profit gains



(b) Expected Profit Gains with strategic and non-strategic customers
(black=strategic population, grey=non-strategic population)
ordered by increasing profit gains for the non-strategic case



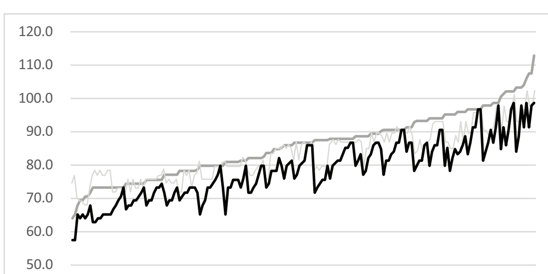
(c) Average Profit Gains varying market uncertainty and fixed costs
(fixed costs 30k-70k, darker bars for higher uncertainty)



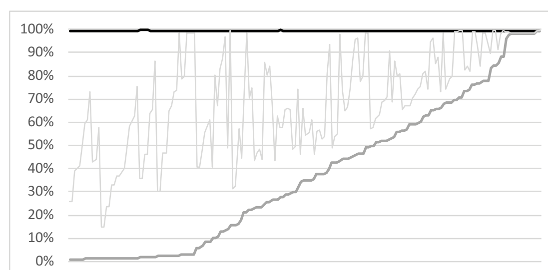
(d) Expected Profit as a function of market uncertainty
(T= Threshold Disocunting, P= Price Discrimination, C= Closure)

Profit gains are measured as the percentage (a, c) or absolute (b) increase in profit when employing threshold discounting compared to the best between price discrimination and closure. Market uncertainty refers to the standard deviation of the market size density function, g. Figure d: the scenario represented has $c_F = 70000$, $\bar{v} = 210$, $\underline{v} = 0$, $\gamma = 1$.

FIGURE 18.1: Profit gains of Threshold Discounting over Traditional Approaches; 216 Scenarios Simulated



(a) Mediated Threshold Discounting: optimal slow period price
(pale grey=SC optimum, dark grey=firm decides, black=intermediary decides)
ordered by increasing firm price



(b) Mediated Threshold Discounting: probability of deal being active
(pale grey=SC optimum, dark grey=firm decides, black=intermediary decides)
ordered by increasing Prob. of an active deal, when the firm chooses the deal

For the activation threshold we report the probability that the deal is active instead of the actual threshold in order to better compare all the scenarios even when different demand distributions are employed.

FIGURE 18.2: Comparison between the equilibrium pricing (left) and activation threshold (right) decisions in Mediated Threshold Discounting, when the choice is made by the service provider and by the intermediary, across the 216 scenarios simulated

outperforms advance selling 72% of the times, with average profit improvement of 7.1%. This is higher when customers are strategic (8%) compared to when they are not (6.3%). The reason is that advance selling removes rationing risk on customers (assuming the firm does not overbook), thus removing all strategic elements to customer visit decision–which we have shown to be in many cases beneficial to the firm. Hence, strategic customer behavior under threshold discounting accounts on average for about 1.5-2% of the profit, and improves profit gains with respect to advance selling by 27% on average (from 6.3% to 8%).

19. Discussion

This paper expands and complements our findings from chapter two. We show that threshold discounting offers outperform traditional approaches, and are therefore best employed, when a firm experiences frequent capacity shortages in peak periods due to seasonal demand, while they should not be used by firms that experience consistently low demand, like new ventures or struggling businesses. Furthermore, we find that threshold discounting offers are less effective when customers exhibit high transaction costs, or when they are offered through a powerful intermediary that can impose the deal specifications, leading in many cases to deals that are active too often and that offer too high a discount compared to what would be optimal for the service provider. The surprising result in Marinesi et al. (2013) regarding the beneficial role of customers has been mostly confirmed by our analysis, but the concavity of the effect suggests that it may be more beneficial for the firm to have a less than fully strategic population.

Interestingly, our analysis also provides two plausible explanations for why Groupon and its major competitors may have discontinued threshold discounting offers: a first one based on the lack of fit between the operational benefit of threshold discounting and the needs of the (demand-starved) firms featured in daily deal websites, and a second one based on the misalignment of incentives along the supply chain, as mentioned above. These two explanations may be two consecutive chapters from the same story: the high discounts and low thresholds used by daily deal websites due to incentive misalignment may have progressively discouraged firms with substantial seasonal demand from running threshold discounting deals, making more room for more demand-starved firms that had no excess demand to smooth–at which point, discontinuing threshold discounting became just the most logical consequence. Finally, using real-world data, we estimate that threshold discounting schemes, if employed correctly, can improve profit by up to 28% (7% on average) compared to traditional capacity management strategies.

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Appendix A. Proofs for Part 1

A.1. **Additional results for Section 3.**

Lemma 5. *In a voting system with purchasing discount to advise on development decisions, all customers vote according to the same strategy, where they cast a vote iff their valuation is higher than a threshold.*

Suppose that at equilibrium k different groups of customers use k different voting strategies $V_D^{(1..k)}$, representing the set of valuations for which a customer of a given group casts a vote. Then, there exists a valuation realization x' such that at least 1 but no more than $k-1$ groups vote, and the firm develops the product. For each of these voting customers to be better off voting, it must be $\delta_D P_D + c_v < x'$ and $P_D(1-\delta_D) > c_v$. If so, each of the other non-voting customers is better off voting. Hence, at equilibrium, customers must employ the same voting strategy, and the firm develops the product iff all of them vote. Given that for every $x > \delta_D P_D + c_v$ customers are collectively better off voting than not voting (and the firm makes profit), their Pareto-dominant strategy is of a threshold-type.

Lemma 6. *In a voting system with purchasing discounts to advise on development decisions, expected sales are always lower than in a traditional no-voting system.*

From the quasi-concavity of $\Pi_N(P_N)$ it follows that $\bar{F}(P) - f(P)(P-c) > 0 \iff P < P_N^*$, which implies that $\delta_D P_D \le P_N^* - c_v \Rightarrow \bar{F}(\delta_D P_D + c_v) - f(\delta_D P_D + c_v)(\delta_D P_D - c - c_F) > 0$; it follows that $\delta_D^* P_D^* + c_v > P_N^*$, hence $\bar{F}(\delta_D^* P_D^* + c_v) < \bar{F}(P_N^*)$.

A.2. ***Proof of Lemma1.*** *Let $v \in \{0,1\}$ represent the voting decision taken by a customer, where 1 stands for voting. Starting from the last action of the game, the equilibrium buying strategy (when the product is developed) is to buy iff $x - P_D + v(1-\delta_D)P_D \ge 0$.*

The firm's development strategy is $D_D^{\nu_o *}(\delta_D, P_D \mid \bar{x}_D^*) = 1$ iff $\pi_D^{\nu_o}(\delta_D, P_D \mid \bar{x}_D^*) \ge 0$, that is, to develop the product if and only if the expected profit-to-go is positive, where the expected subgame profits are given by

$$\pi_D^1(\delta_D, P_D \mid \bar{x}_D^*) = \delta_D P_D - c - c_F$$

$$\pi_D^0(\delta_D, P_D \mid \bar{x}_D^*) = \left[\frac{F(\bar{x}_D^*) - F(P_D)}{\bar{F}(\bar{x}_D^*)}\right]^+ (P_D - c) - c_F$$

with $[y]^{+} = max\,(0, y)$. *Any deviation by this strategy is obviously going to harm the firm. Customer voting strategy* $\bar{x}_D^{*}\,(\delta_D, P_D \mid D_D^{\nu_o *})$ *at equilibrium requires that*

$$\text{(A.1)} \qquad \forall x \geq \bar{x}_D^{*} \begin{cases} (x-\delta_D P_D)\, D_D^{1*}(\delta_D,P_D) - c_v \geq 0 \\ \quad and \\ P_D\,(1-\delta_D) D_D^{1*}(\delta_D,P_D)\, - c_v \geq 0 \end{cases} \qquad \forall x < \bar{x}_D^{*} \begin{cases} (x-\delta_D P_D)\, D_D^{0*}\,(\delta_D, P_D) - c_v < 0 \\ \quad or \\ P_D\,(1-\delta_D) D_D^{0*}(\delta_D,P_D)\, - c_v < 0 \end{cases} .$$

Both conditions in the former set must hold for voters to be better off voting, and at least one condition in the latter set must hold for non-voters to be better off not voting. These imply that $\bar{x}_D^{*} \geq \delta_D P_D + c_v$, *and also that* $D_D^{1*}\,(\delta_D, P_D \mid \bar{x}_D^{*}) = 1$ *and* $D_D^{0*}\,(\delta_D, P_D \mid \bar{x}_D^{*}) = 0$. *From these last two conditions, since* $\pi_D^{\nu_o}\,(\delta_D, P_D \mid \bar{x}_D^{*})$ *is increasing in* $\bar{x}_D^{*}$, *we obtain that it must be* $\bar{x}_{D,1} \leq \bar{x}_D^{*} < \bar{x}_{D,0}$, *where* $\bar{x}_{D,1}(\delta_D, P_D) = min\{\bar{x}_D : \pi_D^{1}(\delta_D, P_D \mid \bar{x}_D) = 1\}$, $\bar{x}_{D,0}\,(\delta_D, P_D) = max\,\{\bar{x}_D : \pi_D^{0}\,(\delta_D, P_D \mid \bar{x}_D) = 0\}$. *Hence, the set of potential equilibrium voting strategies* $\mathcal{X}_D$ *for any given initial announcement of the firm* (δ_D, P_D) *is given by*

$$\mathcal{X}_D\,(\delta_D, P_D) = [max\,\{\bar{x}_{D,1}\,(\delta_D, P_D)\,,\, \delta_D P_D + c_v\}\,,\, \bar{x}_{D,0}\,(\delta_D, P_D))$$

.

It can be shown that customer surplus is the highest for the lowest threshold, which is either $\delta_D P_D + c_v$ *or* $\bar{x}_{D,1}$. *Since the highest profit* $\pi^{1}\,(\delta_D, P_D \mid \bar{x}_D)$ *is already achieved for* $\bar{x}_D = \delta_D P_D$, *it follows that* $\bar{x}_{D,1} < \delta_D P_D + c_v$, *hence the optimal voting strategy is* $\bar{x}_D^{*} = \delta_D P_D + c_v$, *the firm's ex-ante profit function is* $\Pi_D\,(\delta_D, P_D) = \bar{F}\,(\delta_D P_D + c_v)\,(\delta_D P_D - c - c_F)$, *which is quasi-concave in* $\delta_D P_D$, *and the rest follows.*

A.3. ***Proof of Theorem 1.*** *For Point 1, using the Envelope Theorem, we can write* $\frac{d}{dc}\Pi_N^{*} = \frac{\partial}{\partial c}\Pi_N^{*} = -\bar{F}\,(P_N^{*}) < -\bar{F}\,(\delta_D^{*} P_D^{*} + c_v) = \frac{\partial}{\partial c}\Pi_D^{*} = \frac{d}{dc}\Pi_D^{*}$ *where the inequality comes from Lemma 6, and the result is proven since* $\frac{d}{dc_F}\,(\Pi_D^{*} - \Pi_N^{*}) \geq 0$. *Point 2 comes from* $\frac{d}{da}\Pi_N^{*} = \frac{(b+a-c)(a-b+c)}{8a^2}$ *and* $\frac{d}{da}\Pi_D^{*} = \frac{(b+a-c-c_v-c_F)(a-b+c+c_v+c_F)}{8a^2}$ *with simple algebraic manipulations.*

A.4. ***Proof of Theorem 2.*** *Suppose an informative equilibrium exists, and let* x^{-} *be the lowest valuation for which customers vote. The payoff of a customer with valuation* x^{-} *is then* $x^{-} - \delta_P^{*} P_P^{1*}\,(\delta_P^{*}) - c_v$, *which is always negative because* $P_P^{1*}\,(\delta_P^{*}) \geq \frac{x^{-}}{\delta_P^{*}}$: *hence this cannot be an equilibrium because not voting is a strictly better action for her than voting.*

A.5. ***Proof of Theorem 3.*** *The buying strategy is only a function of game history, i.e. to buy iff* $x - P_B^{\nu_o} + v\left(1-\delta_D\right)P_B^{\nu_o} \geq 0$. *The optimal pricing decisions, once the voting outcome* ν_o *is observed, are*

$$P_B^{1*}\left(\delta_B, \bar{P}_B \mid \bar{x}_B^*\right) = \left[\arg\max_{P_B^1}\left(\left[\frac{\bar{F}\left(\delta_B P_B^1\right)}{\bar{F}\left(\bar{x}_B^*\right)}\right]^1 \bar{F}\left(\bar{x}_B^*\right)\left(\delta_B P_B^1 - c\right)\right)\right]^{\bar{P}_B}$$

$$P_B^{0*}\left(\delta_B, \bar{P}_B \mid \bar{x}_B^*\right) = \left[\arg\max_{P_B^0}\left(\left[\frac{F\left(\bar{x}_B^*\right) - F\left(P_B^0\right)}{F\left(\bar{x}_B^*\right)}\right]_0^1 \left(P_B^0 - c\right)\right)\right]^{\bar{P}_B}$$

where $[y]_a^b = max\left(a, min\left(b, y\right)\right)$. *From the above it follows that* $P_B^0 \leq P_B^1$. *For customer voting strategy* $\bar{x}_B^*$ *to be an equilibrium strategy, the following must hold:*

$$\text{(A.2)} \qquad \forall x \geq \bar{x}_B^* \begin{cases} \left(x - \delta_B P_B^{1*}\right) - c_v \geq 0 \\ \quad \textit{and} \\ \left(x - \delta_B P_B^{1*}\right) - c_v \geq \left(x - P_B^{1*}\right) \end{cases} \qquad \forall x < \bar{x}_B^* \begin{cases} \left(x - \delta_B P_B^{0*}\right) - c_v < 0 \\ \quad \textit{or} \\ 0 < \left(x - \delta_B P_B^{0*}\right) - c_v < \left(x - P_B^{0*}\right) \end{cases} .$$

When both conditions on the left hold, a customer that observes a valuation greater than $\bar{x}_B$ *and votes gets an overall positive surplus buying the product (first inequality), and this will be higher than what she would get by not voting (second inequality), given that all other customers vote - hence she is better off voting. When at least one of the conditions on the right holds, a customers that observes a valuation lower than* $\bar{x}_B$ *and votes gets either an overall negative surplus from buying the product (first inequality), or a surplus that is lower than what she could get by not voting and buying the product (second inequality), given that all other customers are not voting - hence she is better off not voting.*

Both conditions in the former set must hold for voters to be better off voting, and at least one condition in the latter set must hold for non-voters to be better off not voting. Note also that since $\bar{x}_B^* \geq \delta_B \bar{P}_B + c_v$, *when customers vote any price lower than* $\bar{x}_B^*/\delta_B$ *would result in the same sales but lower margin than* $\bar{x}_B^*/\delta_B$, *hence* $P_B^{1*} = \bar{P}_B$. *For a voting strategy* $\bar{x}_B^*$ *to be incentive-compatible then, we need*

$$\text{(A.3)} \qquad \bar{x}_B^* - \delta_B \bar{P}_B - c_v \geq 0 \qquad \textit{and} \qquad P_B^{0*}\left(\bar{x}_B^*\right)\left(1 - \delta_B\right) < c_v,$$

hence, customer subgame-perfect voting strategy $\bar{x}_B^*\left(\delta_B, \bar{P}_B\right)$ *after a firm initial announcement* $\left(\delta_B, \bar{P}_B\right)$ *is the consumer surplus-maximizing strategy among those voting strategies that satisfy* (A.3), *i.e. which belong to the set*

$$\mathcal{X}_B = \left[\delta_B \bar{P}_B + c_v,\, P_B^{0*^{-1}}\left(\frac{c_v}{1-\delta_B}\right)\right].$$

The firm's initial move $\left(\delta_B^*,\bar{P}_B^*\right)$ *maximizes the expected profit*

$$\arg\max_{\delta_B,\bar{P}_B}\left[\bar{F}\left(\bar{x}_B^*\left(\delta_B,\bar{P}_B\right)\right)\left(\delta_B\bar{P}_B-c\right)+\left[F\left(\bar{x}_B^*\left(\delta_B,\bar{P}_B\right)\right)-F\left(P_B^{0*}\left(\bar{x}_B^*\left(\delta_B,\bar{P}_B\right)\right)\right)\right]\left(P_B^{0*}\left(\bar{x}_B^*\left(\delta_B,\bar{P}_B\right)\right)-c\right)\right]$$

$$s.t.\ \ \delta_B\leq 1-\frac{c_v}{\bar{P}_B}$$

where the constraint comes from (A.2)*. The firm develops the product iff the resulting expected profit is positive.*

The optimization problem that leads to the firm equilibrium first move $\left(\delta_B^*,\bar{P}_B^*\right)$ *can be further simplified into*

$$\underset{\delta_B,\bar{P}_B}{argmax}\left[\bar{F}\left(\delta_B\bar{P}_B+c_v\right)\left(\delta_B\bar{P}_B-c\right)+\left[F\left(\delta_B\bar{P}_B+c_v\right)-F\left(P_B^{0*}\left(\delta_B\bar{P}_B+c_v\right)\right)\right]\left(P_B^{0*}\left(\delta_B\bar{P}_B+c_v\right)-c\right)\right]$$

$$s.t.\qquad P_B^{0*}\left(\delta_B\bar{P}_B+c_v\right)=\frac{c_v}{1-\delta_B}$$

once we take into account the ability of the firm to set the desired threshold, as shown in Section A.7 (see below).

A.6. ***Proof of Theorem 4.*** *Since the pricing decision is postponed,* ρ_{ν_o} *is a scale factor between prices for the different voting outcomes, and we can fix* $\rho_0=1$ *without loss of generality. The optimal buying strategy is to buy iff* $x\geq\hat{x}_R^{\nu_o}$*, with* $\hat{x}_R^{\nu_o}=P_R^{\nu_o}\left(1+v\left(\rho_{\nu_o}-1\right)\right)$*. A viable threshold voting strategy for customers* $\bar{x}_R$ *must be such that customers vote iff* $x\leq\bar{x}_R$*; in fact in this system voting is more attractive when valuation is low than when it is high. It follows that, in equilibrium, we'll have at most four possible scenarios, depending on customer valuation: when it's lower than* $\hat{x}_R^1$ *they will vote and not buy, when it's in* $[\hat{x}_R^1,\bar{x}_R)$ *they will vote and buy, when it's in* $[\bar{x}_R,\hat{x}_R^0)$ *they will not vote and not buy, and when it's at least* $\hat{x}_R^0$ *they will not vote and buy. In order to make deviations not profitable in each case, we must then require*

$$\text{(A.4)}\qquad\begin{cases}\hat{x}_R^1-P_R^1\leq 0\,and\,r-c_v\geq 0\\ \hat{x}_R^1-P_R^1\geq 0\\ r-c_v\leq\left(\rho_1-1\right)P_R^0\,and\,\hat{x}_R^0-P_R^0\geq r-c_v\\ r-c_v\leq 0\end{cases}\Longrightarrow\begin{cases}\hat{x}_R^1=P_R^1\\ r=c_v\\ \rho_1\geq 1\\ \hat{x}_R^0\geq P_R^0\end{cases},$$

where the optimal pricing strategies are

$$P_R^{1*}=\arg\max_P\left(\left[\frac{\bar{F}\left(P\right)-\bar{F}\left(\bar{x}_R^*\right)}{F\left(\bar{x}_R^*\right)}\right]_0^1\left(P-c\right)\right)\qquad and\qquad P_R^{0*}=\arg\max_P\left(\left[\frac{\bar{F}\left(P\right)}{\bar{F}\left(\bar{x}_R^*\right)}\right]^1\left(P-c\right)\right).$$

The first and the fourth conditions in (A.4) *are consistent with the ex-post optimal pricing conditions, since $\hat{x}_R^1 = P_R^1 \le \bar{x}_R$ and $\hat{x}_R^0 \ge P_R^0 \ge \bar{x}_R$. The second and third conditions can be easily complied by the firm in the initial announcement, and pose no constraints on $\bar{x}_R^*$. Hence, there are many voting thresholds $\bar{x}_R$ leading to an informative equilibrium, and the one that arises is the one that maximizes customer surplus. As for the initial announcement, it must be $r = c_v$, while $\rho_1 > 1$ insures that customers are strictly better off not voting when their valuation is higher than $\bar{x}_R^*$.*

A.7. ***Profit reformulation in Section 4.3 (and Proof for Theorem 6.*** *Let's start from Equation* (4.3)*. In a voting system with bounded pricing, the firm can induce any target voting threshold $\bar{x}_t$ by appropriately choosing $(\delta_B, \bar{P}_B)$, under the condition that $c_v < P_B^0(\bar{x}_t)$, which is satisfied for all thresholds of interest as long as c_v is relatively small compared to product value. To do this, it must be $\delta_B \bar{P}_B = \bar{x}_t - c_v$ and $P_B^{0*}(\bar{x}_t) = \frac{c_v}{1-\delta_B}$ so that $\mathcal{X}_B$ is a singleton. From $c_v < P_B^{0*}(\bar{x}_t) < \bar{x}_t$, $\frac{d}{d\delta_B}\frac{c_v}{1-\delta_B} > 0$ and noting that for the highest incentive-compatible discount $\bar{\delta}_B(\bar{x}_t) = \frac{\bar{x}_t - c_v}{\bar{x}_t}$ we have $\frac{c_v}{1-\delta_B} = \bar{x}_t$, it follows that $\forall \bar{x}_t \exists \delta \in [0, \bar{\delta}_B] : P_B^{0*}(\bar{x}_t) = \frac{c_v}{1-\delta_B}$, and $\bar{P}_B$ is consequently adjusted. Also, $\bar{P}_B(\bar{x}_t, \delta_B)$ is lowest for $\bar{\delta}_B$ and $\bar{P}_B(\bar{x}_t, \bar{\delta}_B) = \bar{x}_t > P_B^{0*}(\bar{x}_t)$ so we're done. Note that restricting the firm choice of initial announcement $(\delta_B, \bar{P}_B)$ to those for which $\mathcal{X}_B$ is a singleton does not reduce the firm profit! In fact, take any $\delta_B, \bar{P}_B$ such that $\bar{x}_B^*(\delta_B, \bar{P}_B) > \delta_B \bar{P}_B + c_v$. Then the announcement $(\delta'_B, \bar{P}'_B)$ such that $\bar{x}_B^*(\delta'_B, \bar{P}'_B) = \delta'_B \bar{P}'_B + c_v = \bar{x}_B^*(\delta_B, \bar{P}_B)$ leads to a higher profit, being,*

$$
\begin{aligned}
&\Pi_B(\delta_B, \bar{P}_B) = \\
&= \bar{F}(\bar{x}_B^*(\delta_B, \bar{P}_B))(\delta_B \bar{P}_B - c) + [F(\bar{x}_B^*(\delta_B, \bar{P}_B)) - F(P_B^{0*}(\bar{x}_B^*(\delta_B, \bar{P}_B)))](P_B^{0*}(\bar{x}_B^*(\delta_B, \bar{P}_B)) - c) < \\
&< \bar{F}(\bar{x}_B^*(\delta'_B, \bar{P}'_B))(\delta'_B, \bar{P}'_B - c) + [F(\bar{x}_B^*(\delta'_B, \bar{P}'_B)) - F(P_B^{0*}(\bar{x}_B^*(\delta'_B, \bar{P}'_B)))](P_B^{0*}(\bar{x}_B^*(\delta'_B, \bar{P}'_B)) - c) = \\
&= \Pi_B(\delta'_B, \bar{P}'_B).
\end{aligned}
$$

Equation (4.2) *for bounded pricing is explained by noting that $\bar{x}_B^* = \underset{P}{argmax} \bar{F}_1(P)(P - c)$ always. Suppose not, i.e. $\underset{P}{argmax} \bar{F}_1(P)(P-c) = \bar{x}_B^* + \Delta > \bar{x}_B^*$. Then the firm could choose $(\delta'_B, \bar{P}'_B)$ such that $\delta'_B \bar{P}'_B + c_v = \bar{x}_B^* + \Delta$ and $\frac{c_v}{1-\delta'_B} = P_B^{0*}(\bar{x}_B^* + \Delta)$, resulting in $\Pi_B(\delta'_B, \bar{P}'_B) > \Pi_B^*$. In fact*

$$\bar{F}(\bar{x}_B^* + \Delta)(\delta'_B \bar{P}'_B - c) < \bar{F}(\bar{x}_B^* + \Delta)(\delta_B^* \bar{P}_B^* - c)$$

$$[F(\bar{x}_B^* + \Delta) - F(P_B^{0*}(\bar{x}_B^* + \Delta))](P_B^{0*}(\bar{x}_B^* + \Delta) - c) < [F(\bar{x}_B^*) - F(P_B^{0*}(\bar{x}_B^*))](P_B^{0*}(\bar{x}_B^*) - c).$$

Hence it must be $\bar{x}_B^* = \arg\underset{P}{max} \bar{F}_1(P)(P-c)$, and we can decompose $\bar{F}(\bar{x}_B^*)(\delta_B^* \bar{P}_B^* - c)$ into $\bar{F}(\bar{x}_B^*)(\bar{x}_B^* - c) - \bar{F}(\bar{x}_B^*) c_v$ and the rest follows.

As for a reverse voting system, Equation (4.4) *is the Pareto-dominant voting strategy for the equilibria found in Section A.6, while the profit reformulation in* (4.2) *follows by noting that $P_R^{0*} = \bar{x}_R^*$. Suppose not,*

Chesterfield Sofa

Table Lamp

Figure A.1: Products employed for our numerical study in Section 18

i.e. $P_R^{0*} = \bar{x}_R^* + \Delta > \bar{x}_R^*$. *Then customers could coordinate on the threshold* $\bar{x}_R^* + \Delta$ *and increase their surplus, since this would increase the ex-ante surplus in the low contingency while not decreasing the ex-ante surplus in the high contingency, that is*

$$\left[\bar{F}\left(P_R^{1*}\left(\bar{x}_R^*+\Delta\right)\right)-\bar{F}\left(\bar{x}_R^*+\Delta\right)\right]\left(x-P_R^{1*}\left(\bar{x}_R^*+\Delta\right)\right)\geq\left[\bar{F}\left(P_R^{1*}\left(\bar{x}_R^*\right)\right)-\bar{F}\left(\bar{x}_R^*\right)\right]\left(x-P_R^{1*}\left(\bar{x}_R^*\right)\right)$$

$$\left(x-P_R^{0*}\left(\bar{x}_R^*+\Delta\right)\right)\cdot\bar{F}\left(\bar{x}_R^*+\Delta\right)=\left(x-\bar{x}_R^*-\Delta\right)\cdot\bar{F}\left(\bar{x}_R^*+\Delta\right)=\left(x-P_R^{0*}\left(\bar{x}_R^*\right)\right)\cdot\bar{F}\left(P_R^{0*}\left(\bar{x}_R^*\right)\right)$$

with the profit being strictly higher if S is convex. The rest follows. Note that the ex-post optimal pricing decisions do not depend on the system employed per se, but only on the posterior information, hence the use of the common pricing functions P_h^* and P_l^*. Theorem 6 follows from the definitions of PI_j and CI_j, and from Equations (4.3) *and* (4.4).

A.8. ***Proof of Theorem 5.*** *The first part follows from the definitions of CI_B and CI_R. As for the second part, $f(x)$ differentiable implies that $\Pi_B(\bar{x})$ is also differentiable, with*

$$\frac{d}{d\bar{x}}\Pi_B=-f(\bar{x})(\bar{x}-c_v-c)+\bar{F}(\bar{x})+\left[f(\bar{x})-f(P_0^*(\bar{x}))\frac{d}{d\bar{x}}P_0^*(\bar{x})\right](P_0^*(\bar{x})-c)+ \tag{A.5}$$

$$+[F(\bar{x})-F(P_0^*(\bar{x}))]\frac{d}{d\bar{x}}P_0^*(\bar{x})\,.$$

It can be shown that the subgame profit after a low contingency $\pi^0\left(P_0\mid\bar{x}\right)=\left[F\left(\bar{x}\right)-F\left(P_0\right)\right]\left(P_0-c\right)$ *is quasi-concave in P_0 because f has non-decreasing hazard rate, hence from the f.o.c. we obtain* $\left[F\left(\bar{x}\right)-F\left(P_0^*\right)\right]=f\left(P_0^*\right)\left(P_0^*-c\right)$, *leading to* $\frac{d}{d\bar{x}}\Pi_B=\bar{F}\left(\bar{x}\right)+f\left(\bar{x}\right)\left(P_0^*\left(\bar{x}\right)-\bar{x}+c_v\right)$ *by substitution in* (A.5).

Consider now the shifted distribution $g\left(x+k\right)=f\left(x\right)$, *with $k>0$. Let $P_0^{g*}\left(\bar{x}\right)$ be the optimal subgame price under the low contingency and $\Pi_B^g\left(\bar{x}\right)$ the firm profit under the shifted distribution g. Then we have* $\left[\frac{d}{d\bar{x}}\Pi_B^g\right]_{\bar{x}+k}=\bar{G}\left(\bar{x}+k\right)+g\left(\bar{x}+k\right)\left(P_0^{g*}\left(\bar{x}\right)-\bar{x}-k+c_v\right)=\bar{F}\left(\bar{x}\right)+f\left(\bar{x}\right)\left(P_0^{g*}\left(\bar{x}\right)-\bar{x}-k+c_v\right)$ *and*

$$\left[\frac{d}{d\bar{x}}\Pi_B\right]_{\bar{x}}-\left[\frac{d}{d\bar{x}}\Pi_B^g\right]_{\bar{x}+k}=f\left(\bar{x}\right)\left(P_0^*\left(\bar{x}\right)+k-P_0^{g*}\left(\bar{x}\right)\right)\geq 0. \tag{A.6}$$

The above is never negative because $P_0^*(\bar{x}) + k \geq P_0^{g*}(\bar{x})$ *since* $\frac{d}{dP}\pi^0(P \mid \bar{x}) \geq \frac{d}{dP}\pi^{0,g}(P + k \mid \bar{x} + k)$ *for every* P*. Inequalities are strict when* $f(P_0) > 0$ *hence* S *convex is a sufficient condition. Let* $\bar{x}^f = \arg\max_{\bar{x}} \Pi_B(\bar{x})$, $\bar{x}^g = \arg\max_{\bar{x}} \Pi_B^g(\bar{x})$*. From* (A.6) *we have that* $\forall \bar{x} \geq \bar{x}^f$, $\Pi_B(\bar{x}^f) \geq \Pi_B(\bar{x}) \Rightarrow \Pi_B^g(\bar{x}^f + k) \geq \Pi_B(\bar{x} + k)$*, hence* $\bar{x}^g \leq \bar{x}^f + k$*, which implies* $\bar{F}(\bar{x}^f) \leq \bar{G}(\bar{x}^g)$*. The argument is identical for* $k < 0$*, the conclusion being reversed.*

The Traditional Approach.

B.1. **Equilibrium outcome.** Under seasonal closure, the service provider offers his services only in the hot period; hence, customers visit in that period and the firm's expected profit is given by (**??**).

Under price discrimination, the firm charges r_s during the slow period. Take any price r_s charged in the slow period, and let the visit strategy of all customers except customer i be represented by the set $\mathcal{V}_{-i}(r_s) \subseteq [\underline{v}, v_h)$, so that a customer $j \neq i$ visits in the slow period if $v_s \in \mathcal{V}_{-i}(r_s)$ and visits in the hot period otherwise. Then, the expected incremental surplus of customer i with valuation $v_{s,i}$ from visiting in the slow period rather than in the hot period is given by

$$(v_h - r_h) \int_0^{+\infty} \min\left(1, \frac{k}{\hat{\alpha}_p x}\right) dG_c - (v_{s,i} - r_s) \int_0^{+\infty} \min\left(1, \frac{k}{(1-\hat{\alpha}_p)\, x}\right) \mathrm{d}G_c(x),$$

where $\hat{\alpha}_p = 1 - \int_{\mathcal{V}_{-i}(r_s)} dH$ is the fraction of the population that visits in the hot period. Note that the above expression is increasing in $v_{s,i}$, hence customer i visit strategy is of a threshold type, such that she visits during the slow period iff her valuation for the slow period is high enough. It follows that all customers in equilibrium use a threshold subscription strategy. Given that customers are all homogenous ex-ante, their threshold visit strategy will be to visit in the slow period iff $v_s > \hat{v}_p(r_s)$, with $\hat{v}_p(r_s)$ being the valuation for which a customer is indifferent between the two periods, given by

$$(v_h - r_h) \int_0^{+\infty} \min\left(1, \frac{k}{H(\hat{v}_p(r_s))\, x}\right) dG_c(x) - (\hat{v}_p(r_s) - r_s) \int_0^{+\infty} \min\left(1, \frac{k}{\bar{H}(\hat{v}_p(r_s))\, x}\right) \mathrm{d}G_c(x) = 0.$$

The visit strategy $\hat{v}_p(r_s)$ is unique because the LHS of the above expression is strictly decreasing in $\hat{v}_p$. The firm optimal price choice is then the one that maximizes expected profit. In equilibrium we have that

$$\text{(B.1)} \quad \Pi_p^* = \arg\max_{r_s} (r_h - c) \int_0^{+\infty} \min(k, H(\hat{v}_p(r_s))\, x)\, \mathrm{d}G(x) + (r_s - c) \int_0^{+\infty} \min\left(k, \bar{H}(\hat{v}_p(r_s))\, x\right) \mathrm{d}G(x) - 2c_F.$$

B.2. **Additional lemma.**

Lemma 7. *Under price discrimination, the expected margin of the firm M_p increases in the price r_s, and the expected capacity utilization U_p is maximized for $r_s = H^{-1}\left(\hat{v}_t^{-1}(1/2)\right)$, where M_p and U_p are defined as*

$$M_p(r_s) = \frac{\int_0^{+\infty} \left[\min(k, H(\hat{v}_p(r_s))\, x)(r_h - c) + \min\left(k, \left(\bar{H}(\hat{v}_p(r_s))\right) x\right)(r_s - c)\right] \mathrm{d}G(x)}{\int_0^{+\infty} \left[\min(k, H(\hat{v}_p(r_s))\, x) + \min\left(k, \left(\bar{H}(\hat{v}_p(r_s))\right) x\right)\right] \mathrm{d}G(x)}$$

$$U_p(r_s) = \int_0^{+\infty} \left[\min\left(1, k^{-1} H(\hat{v}_p(r_s))\, x\right) + \min\left(1, k^{-1} \bar{H}(\hat{v}_p(r_s))\, x\right)\right] \mathrm{d}G(x)$$

For M_p, it is convenient to define

$$\eta\left(r_s\right)=\frac{\int_0^{+\infty}\min\left(H(\hat{v}_p\left(r_s\right))\,x,k\right)\mathrm{d}G(x)}{\int_0^{+\infty}\min\left(H(\hat{v}_p\left(r_s\right))\,x,k\right)\mathrm{d}G(x)+\int_0^{+\infty}\min\left(\bar{H}(\hat{v}_p\left(r_s\right))\,x,k\right)\mathrm{d}G(x)}$$

as the (expected) fraction of sales served during the hot period; then we have $M_p\left(r_s\right) = \eta\left(r_h-c\right) + (1-\eta)\left(r_s-c\right)$ and $\frac{d}{dr_s}M_p = \eta'\left(r_s\right)\left(r_h-r_s\right)+(1-\eta)$, and the result follows by noting that $\eta'\left(r_s\right) > 0$ because $\frac{d}{dr_s}H(\hat{v}_p\left(r_s\right)) > 0$, and $\frac{d}{dr_s}\min\left(x\bar{H}(\hat{v}_p\left(r_s\right)),k\right) \leq 0$, strictly if $x\bar{H}(\hat{v}_p\left(r_s\right)) \leq k$.

For U_p, it suffices to note that the integrand function $\min\left(1,k^{-1}H(\hat{v}_p(r_s))\,x\right)+\min\left(1,k^{-1}\bar{H}(\hat{v}_p(r_s))\,x\right)$ is increasing in r_s if $H(\hat{v}_p(r_s)) < \frac{1}{2}$ and decreasing in r_s if $H(\hat{v}_p(r_s)) > \frac{1}{2}$ for every x (strictly so iff $\bar{H}(\hat{v}_p(r_s))\,x > k$ and $H(\hat{v}_p(r_s))\,x > k$ respectively).

B.3. **Lemma 2.** Let $\pi_p\left(r_s^p,x\right) = \left(r_h-c\right)\min\left(k,xH\left(\hat{v}_p\left(r_s^p\right)\right)\right) + \left(r_s^p-c\right)\min\left(k,x\bar{H}\left(\hat{v}_p\left(r_s^p\right)\right)\right) - 2c_F$ and $\pi_c\left(x\right) = \left(r_h-c\right)\min\left(k,x\right) - c_F$ be the profit earned by the service provider when market size realization is equal to x and he opens or close on the hot period, respectively, with r_s^p being the solution to (B.1). The single crossing of the two functions follows by noting that $\pi_p\left(r_s^p,0\right) = -2c_F < -c_F = \pi_c\left(0\right)$, $\frac{d}{dx}\pi_c(x) > \frac{\partial}{\partial x}\pi_p\left(r_s^p,x\right) > 0$ for $x<k$, $\frac{d}{dx}\pi_c(x) = 0 < \frac{\partial}{\partial x}\pi_p\left(r_s^p,x\right)$ for $x \in \left(k,k\left(xH(\hat{v}_p(r_s^p))\right)^{-1}\right)$, and that $\lim_{x\to+\infty}\pi_p\left(r_s^p,x\right) = k\left(r_h+r_s^p-2c\right)-2c_F > k\left(r_h-c\right)-c_F = \lim_{x\to+\infty}\pi_c\left(x\right)$. This also implies that market realization x° for which $\pi_p(r_s^p,x^\circ) = \pi_c(x^\circ)$ must be higher than k (Figure 9.2). $\frac{d}{dc_F}x^\circ > 0$ and $\frac{d}{dc}x^\circ > 0$ come from $\pi_p\left(r_s^p,x\right)-\pi_c\left(x\right)$ being strictly decreasing in c_F, and also in c $\forall x>k$.

Threshold discounting.

B.4. **Equilibrium outcome in the Customer Continuation Game $\hat{\Gamma}\left(r_s,n\right)$.**

B.4.1. *Proving threshold visit and subscription strategies.* We analyze customer strategies proceeding backward, beginning from the visit strategy. If the deal is not active, the firm closes on the slow period and everyone visits in the hot period since $v_h > r_h$ and the probability of being served is always positive. If the deal is active, customer i visits in the period in which she expects to make the higher surplus. Specifically, let $v_{s,i}$ be the slow period valuation of customer i, and let $s_i = 1$ if customer i previously subscribed and $s_i = 0$ otherwise; let the visit strategy of customer i be represented by the set of valuations for which customer i visits in the slow period, $\nu_i = \nu_i\left(r_s,n,s_i\right)$, and let $\nu_{-i}\left(r_s,n,s_{-i}\right)$ be the vector containing the visit strategies of all customers except i. Let also σ_{-i} be the vector of subscription strategies $\sigma_j\left(r_s,n\right)$ of all customers except i, such that customer j subscribes to the deal iff $v_{s,j} \in \sigma_j\left(r_s,n\right)$. Then $v_{s,i} \in \nu_i$ iff

(B.2)

$$\left(v_{s,i}-s_i\,r_s-(1-s_i)\,r_h\right)\frac{1}{Pr\left(x\in\mathcal{A}\right)}\int_{\mathcal{A}}\min\left(1,k\left(\bar{\alpha}_v(\nu_{-i}(r_s,n,s_{-i}))\,x\right)^{-1}\right)\mathrm{d}G_c\left(x\right)>$$

$$>(v_h-r_h)\,\frac{1}{Pr\,(x\in\mathcal{A})}\int_{\mathcal{A}}\min\left(1,k\left(\alpha_v(\nu_{-i}(r_s,n,s_{-i}))\,x\right)^{-1}\right)\mathrm{d}G_c\,(x)\,,$$

where $\alpha_v\,(\nu_{-i})$ is the fraction of customers that visit in the hot period given their visit strategy ν_{-i}, $\bar{\alpha}_v\,(\nu_{-i})=1-\alpha_v\,(\nu_{-i})$, and where $\mathcal{A}=\left\{x:\,x\int_{\sigma_{-i}(r_s,n)}\mathrm{d}H\,(v)\geq n\right\}$ is the set of all market states in which the deal is active, that is, when at least n customers subscribed. Note that the LHS of (B.2) is increasing in $v_{s,i}$ and the RHS does not depend on $v_{s,i}$, implying that all customers use the same threshold visit strategy in which they visit in the slow period iff $v_s>\hat{v}_t\,(r_s,n)$ if they subscribed and iff $v_s>\hat{v}_{nt}\,(r_s,n)$ if they did not subscribe, with $\hat{v}_t\,(r_s,n)<\hat{v}_{nt}\,(r_s,n)$ being the slow-period valuation of the customer that is indifferent between the two periods in the two cases. Note that information incompleteness in the game is represented by the uncertain market state x, and the belief about x is computed for each player at each information set according to Bayes' rule, taking into account the strategy of the other players.

As for the subscription strategy, a customer subscribes iff doing so increases her expected future payoff, i.e. iff

(B.3)

$$Pr(x\in\mathcal{A})\left[(v_{s,i}-r_s)\int_{\mathcal{A}}\min\left(1,\frac{k}{\bar{\alpha}_v(\hat{v}_t,\hat{v}_{nt},\sigma_{-i})\,x}\right)\mathrm{d}G_c\,(x)-(v_h-r_h)\int_{\mathcal{A}}\min\left(1,\frac{k}{\alpha_v(\hat{v}_t,\hat{v}_{nt},\sigma_{-i})\,x}\right)\mathrm{d}G_c\,(x)\right]>0.$$

If $Pr\,(x\in\mathcal{A})>0$, then the expected gain from subscribing is strictly increasing in $v_{s,i}$, hence subscribing is also a threshold strategy; if instead $Pr\,(x\in\mathcal{A})=0$, then the deal is never active, meaning that customers never subscribe, which is also a threshold strategy with threshold v_h.

B.4.2. *Multiple equilibria in the Customers Continuation Game.* Equilibrium strategies for a Perfect Bayesian Equilibrium are defined by the behavior strategy combination $(\hat{v}_t,\hat{v}_{nv},\hat{v}_{sv})$, i.e. by a vector specifying customer threshold strategies, where $\hat{v}_t$ is a subscription strategy that specifies the minimum slow-period valuation above which a customer subscribes, and $\hat{v}_{nv}$ $(\hat{v}_{sv})$ is the visit strategy that, contingent on the deal being active, specifies the minimum slow-period valuation above which a customer visits in the slow period, given that she previously did not subscribe (did subscribe) to the deal. No visit strategy needs to be specified in case the deal is not active, since all customers visit in the hot period.

We divide equilibria of this game into two types. Type I equilibria are those where customers subscribe with some positive probability, i.e. $\hat{v}_t^I<v_h$, and type II equilibria are those where customers never subscribe, i.e. $\hat{v}_t^{II}=v_h$. Specifically, type II equilibria are characterized by the behavior strategy combination $\left(\hat{v}_t^{II},\hat{v}_{nv}^{II},\hat{v}_{sv}^{II}\right)$ such that $\hat{v}_t^{II}=v_h$, with the other strategies having no impact since in equilibrium the deal is never active. We rule out these equilibria because they do not exist in the more general case, in which the population of customers comprises of both strategic and myopic customers, as studied in

Section 17.1. Furthermore, type II equilibria are Pareto dominated by type I equilibria: this will be shown in subsubsection B.4.6, once customer strategy for type I equilibria has been appropriately defined.

B.4.3. *Type I equilibria.* Henceforth, we'll refer to strategies of type I equilibria without the superscript I. In type I equilibria, the visit strategy when a customer has subscribed, $\hat{v}_{sv}$, is given by

(B.4)
$$(\hat{v}_{sv}-r_s)\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty}\min\left(1,k\left(\bar{\alpha}_t(\hat{v}_t,\hat{v}_{nv},\hat{v}_{sv})\,x\right)^{-1}\right)\mathrm{d}G_c(x)\;-\;(v_h-r_h)\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty}\min\left(1,k\left(\alpha_t(\hat{v}_t,\hat{v}_{nv},\hat{v}_{sv})\,x\right)^{-1}\right)\mathrm{d}G_c(x)=0,$$

where $\alpha_t(\hat{v}_t,\hat{v}_{nv},\hat{v}_{sv})=\bar{H}\left(\max\left(\hat{v}_t,\hat{v}_{sv}\right)\right)+\left[H\left(\hat{v}_t\right)-H\left(\hat{v}_{nv}\right)\right]^+$ is the fraction of customers visiting in the hot period, and ,$\bar{\alpha}_t=1-\alpha_t$. The visit strategy when a customer has not previously subscribed is given by

$$(\hat{v}_{nv}-r_h)\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty}\min\left(1,k\left(\bar{\alpha}_t(\hat{v}_t,\hat{v}_{nv},\hat{v}_{sv})\,x\right)^{-1}\right)\mathrm{d}G_c(x)-(v_h-r_h)\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty}\min\left(1,k\left(\alpha_t(\hat{v}_t,\hat{v}_{nv},\hat{v}_{sv})\,x\right)^{-1}\right)\mathrm{d}G_c(x)=0,$$

which implies that $\hat{v}_{nv}>\hat{v}_{sv}$. The subscription strategy is given by

(B.5)
$$(\hat{v}_t-r_s)\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty}\min\left(1,k\left(\bar{\alpha}_t(\hat{v}_t,\hat{v}_{nv},\hat{v}_{sv})\,x\right)^{-1}\right)\mathrm{d}G_c(x)\;-\;(v_h-r_h)\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty}\min\left(1,k\left(\alpha_t(\hat{v}_t,\hat{v}_{nv},\hat{v}_{sv})\,x\right)^{-1}\right)\mathrm{d}G_c(x)=0.$$

Equations (B.4) and (B.5) are the same mathematical form, yet $\hat{v}_{sv}$ and $\hat{v}_t$ may in principle differ, since such equations may allow for multiple solutions. However, this is never the case. In fact, note that the case $\hat{v}_t<\hat{v}_{sv}$ is not possible, because $\hat{v}_{nv}>\hat{v}_{sv}\Rightarrow\hat{v}_{nv}>\hat{v}_t$ implies that $\alpha_t=\alpha_t\left(\hat{v}_{sv},\hat{v}_{nv}\right)$, hence if (B.4) holds then the LHS of (B.5) is negative, leading to a contradiction. The case $\hat{v}_t>\hat{v}_{sv}$ is also not possible since it implies $\alpha_t=\alpha_t\left(\hat{v}_t,\hat{v}_{nv}\right)$ hence if (B.5) holds then the LHS of (B.4) is negative, leading to a contradiction. It follows that $\hat{v}_t=\hat{v}_{sv}$, i.e. when the deal is active, customers visit in the slow period iff they have previously subscribed. Hence, in a type I equilibrium subscribers will always visit in the slow period and non-subscribers will always visit in the hot period, and the subscription threshold $\hat{v}_t$ fully characterizes customer behavior.

B.4.4. *Uniqueness of type I equilibria in the Customer Continuation Game when $r_s\geq\bar{r}$.* Equation (B.5) can be conveniently rewritten as

$$\text{(B.6)}\qquad\frac{\hat{v}_t-r_s}{v_h-r_h}=\frac{\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty}\min\left(1,k\left(H\left(\hat{v}_t\right)x\right)^{-1}\right)\mathrm{d}G_c(x)}{\int_{n\bar{H}(\hat{v}_t)^{-1}}^{+\infty}\min\left(1,k\left(\bar{H}\left(\hat{v}_t\right)x\right)^{-1}\right)\mathrm{d}G_c(x)}.$$

Let the LHS and RHS of (B.6) be henceforth referred to as $LHS_t\left(\hat{v}_t,r_s\right)$ and $RHS_t\left(\hat{v}_t,n\right)$. Then equation (B.6) has a unique solution $\hat{v}_t$ for every $r_s\geq\bar{r}$ and every n, because $LHS_t\left(\hat{v}_t,r_s\right)$ strictly increases in $\hat{v}_t$ and $RHS_t\left(\hat{v}_t,n\right)$ weakly decreases in $\hat{v}_t$. To see why, first note that all integrands at the numerator of $RHS_t\left(\hat{v}_t,n\right)$

decrease in $\hat{v}_t$ and all integrands at the denominator of $RHS_t(\hat{v}_t, n)$ increase in $\hat{v}_t$. It follows that $RHS_t(\hat{v}_t, n)$ decreases in $\hat{v}_t$ because $n\bar{H}(\hat{v}_t)^{-1}$ increases in $\hat{v}_t$ and $\min(1, k\,(H(\hat{v}_t)\,x)^{-1})\min(1, k\left(\bar{H}(\hat{v}_t)\,x\right)^{-1})^{-1}$ decreases in x, due to the property that given $a_1...a_n, b_1...b_n > 0$ such that $a_i/b_i \geq a_{i+1}/b_{i+1}\forall i = 1..n-1$ we have that $\sum_{n_1}^{n} a_j / \sum_{n_1}^{n} b_j \geq \sum_{n_2}^{n} a_j / \sum_{n_2}^{n} b_j \,\forall n_1 < n_2 \leq n$. Hence, when $r_s \geq \bar{r}$, there exists a unique type I equilibrium in the customer continuation game that follows, characterized by (B.6).

B.4.5. *Optimal firm announcement.* The optimal firm announcement (r_s^t, n^t) is given by

$$\text{(B.7)}\qquad r_s^t, n^t = \arg\max_{r_s, n} \int_0^{n\bar{H}(\hat{v}_t(r_s,n))^{-1}} \left[(r_h - c)\min(x,k) - c_F\right] \mathrm{d}G(x) +$$

$$+ \int_{n\bar{H}(\hat{v}_t(r_s,n))^{-1}}^{+\infty} \left[(r_h - c)\min(xH(\hat{v}_t(r_s,n)),k) + (r_s - c)\min\left(x\bar{H}(\hat{v}_t(r_s,n)),k\right) - 2c_F\right] \mathrm{d}G(x)\,.$$

We now show that the firm is always better off announcing a discounted price $r_s^t > \bar{r}$, where $\bar{r} = H^{-1}\left(\frac{1}{2}\right) - v_h + r_h$. Let $LHS_t(\hat{v}_t, r_s)$ and $RHS_t(\hat{v}_t, n)$ be the LHS and RHS of (B.6), as before. First, note that $LHS_t\left(H^{-1}\left(\frac{1}{2}\right), \bar{r}\right) = 1$, and also that $r_s^t \geq \bar{r} \Rightarrow \hat{v}_t(r_s^t, n) \geq H^{-1}\left(\frac{1}{2}\right) \forall n$. In fact, suppose not, then $\hat{v}_t < H^{-1}\left(\frac{1}{2}\right) \Rightarrow RHS_t(\hat{v}_t, n) \geq 1$ and $LHS_t(\hat{v}_t, r_s^t) < 1$ which violates (B.6). Now suppose that the solution to (B.7) is such that $r_s^t < \bar{r}$. Then the service provider can earn a higher profit by choosing the deal $(\bar{r}, \bar{n}_t)$ where $\bar{n}_t = \frac{1}{2}\frac{n^t}{\bar{H}(\hat{v}_t(r_s^t, n^t))}$, in fact

$$\Pi_t\left(r_s^t, n^t\right) = \int_0^{\frac{n^t}{\bar{H}(\hat{v}_t(r_s^t,n^t))}} \left[(r_h - c)\min(x,k) - c_F\right] \mathrm{d}G(x) +$$

$$+ \int_{\frac{n^t}{\bar{H}(\hat{v}_t(r_s^t,n^t))}}^{+\infty} \left[(r_h - c)\min\left(H\left(\hat{v}\left(r_s^t, n^t\right)\right)x, k\right) + \left(r_s^t - c\right)\min\left(\bar{H}\left(\hat{v}\left(r_s^t, n^t\right)\right)x, k\right) - 2c_F\right] \mathrm{d}G(x) \leq$$

$$\leq \int_0^{\frac{n^t}{\bar{H}(\hat{v}_t(r_s^t,n^t))}} \left[(r_h - c)\min(x,k) - c_F\right] \mathrm{d}G(x) + \int_{\frac{n^t}{\bar{H}(\hat{v}_t(r_s^t,n^t))}}^{+\infty} \left[(r_h - c)\min\left(\frac{x}{2},k\right) + (\bar{r} - c)\min\left(\frac{x}{2},k\right) - 2c_F\right] \mathrm{d}G(x) = \Pi_t(\bar{r}, \bar{n}_t)\,,$$

where the inequality holds because when the deal is active both capacity utilization and average margin are higher for any market size realization equal to or above $n^t/\bar{H}(\hat{v}_t(r_s^t, n^t))$, and the last equality holds because $\bar{n}_t/\bar{H}(\hat{v}_t(\bar{r}, \bar{n}_t)) = 2\bar{n}_t = n^t/\bar{H}(\hat{v}_t(r_s^t, n^t))$. The result is then proven by contradiction.

B.4.6. *Pareto Dominance of Type I equilibria over Type II.* Let $CS^I(v_s)$ and $CS^{II}(v_s)$ be the expected surplus for a customer with slow-period valuation v_s under type I and type II equilibria, respectively. Let

$\alpha_t = H(\hat{v}_t(r_s, n))$ and $\bar{\alpha}_t = 1 - \alpha_t$. Then for any initial announcement (r_s, n) we have

$$CS^I(v_s; v_s > \hat{v}_t(r_s,n)) = (v_h - r_h)\int_0^{n\bar{\alpha}_t^{-1}} \min\left(1, k\,x^{-1}\right) \mathrm{d}G_c(x) + (v_s - r_s)\int_{n\bar{\alpha}_t^{-1}}^{+\infty} \min\left(1, k\left(\bar{\alpha}_t x\right)^{-1}\right) \mathrm{d}G_c(x) >$$

$$= (v_h - r_h)\int_0^{n\bar{\alpha}_t^{-1}} \min\left(1, k\,x^{-1}\right) \mathrm{d}G_c(x) + (v_h - r_h)\int_{n\bar{\alpha}_t^{-1}}^{+\infty} \min\left(1, k\left(\alpha_t x\right)^{-1}\right) \mathrm{d}G_c(x) \geq$$

$$> (v_h - r_h)\int_0^{+\infty} \min\left(1, k\,x^{-1}\right) \mathrm{d}G_c(x) = CS^{II}(v_s; v_s > \hat{v}_t(r_s,n)),$$

where the first inequality comes from (B.6), and

$$CS^I(v_s; v_s \leq \hat{v}_t(r_s,n)) = (v_h - r_h)\int_0^{n\bar{\alpha}_t^{-1}} \min\left(1, k\,x^{-1}\right) \mathrm{d}G_c(x) + (v_h - r_h)\int_{n\bar{\alpha}_t^{-1}}^{+\infty} \min\left(1, k\left(\alpha_t x\right)^{-1}\right) \mathrm{d}G_c(x) \geq$$

$$\geq (v_h - r_h)\int_0^{+\infty} \min\left(1, k\,x^{-1}\right) \mathrm{d}G_c(x) = CS^{II}(v_s; v_s \leq \hat{v}_t(r_s,n)).$$

B.5. **Lemma 3.** See subsubsection B.4.3.

B.6. **Lemma 4.** See subsubsection B.4.5.

B.7. **Theorem 7.** The proof follows two steps.

B.7.1. *Step 1: The strategic scarcity effect.* First, we show that

(B.8)
$$H(\hat{v}_t(r,n)) < H(\hat{v}_p(r))\ if\ r \geq \bar{r}\ and\ n > 0.$$

To this aim, we compare customer strategy in (B.6) with the visit strategy under price discrimination, rewritten in a similar way here below,

(B.9)
$$\frac{\hat{v}_p - r_s}{v_h - r_h} = \frac{\int_0^{+\infty} \min\left(1, k\left(H(\hat{v}_p)\,x\right)^{-1}\right) \mathrm{d}G_c(x)}{\int_0^{+\infty} \min\left(1, k\left(\bar{H}(\hat{v}_p)\,x\right)^{-1}\right) \mathrm{d}G_c(x)},$$

and let the LHS and RHS be referred to as $LHS_p(\hat{v}_p, r_s)$ and $RHS_p(\hat{v}_p)$, respectively. The condition $r \geq \bar{r}$ implies $\hat{v}_t(r,n) \geq H^{-1}\left(\frac{1}{2}\right)$, which implies that $\min\left(1, k/(H(\hat{v}_t)\,x)\right) / \min\left(1, k/(\bar{H}(\hat{v}_t)\,x)\right)$ decreases in x, which in turns implies that $\int_y^{+\infty} \min\left(1, k/(H(\hat{v}_t)\,x)\right) \mathrm{d}G(x) / \int_y^{+\infty} \min\left(1, k/(\bar{H}(\hat{v}_t)\,x)\right) \mathrm{d}G(x)$ decreases in y, as discussed in subsubsection B.4.4. Then $RHS_p(\hat{v}) > RHS_t(\hat{v}, n)\ \forall n > 0$, and by noting that $LHS_p(r) = LHS_t(r)$, it follows that $\hat{v}_t(r) < \hat{v}_p(r)$, hence the result.

B.7.2. *Step 2: Threshold Discounting outperforms the Traditional Approach.* Suppose that $\Pi_p > \Pi_c$. Now, we show that the previous property implies that $\Pi_t\left(r_s^t, n^t\right) > \Pi_p\left(r_s^p\right)$. In fact,

$$\Pi_p\left(r_s^p\right) = \int_0^{+\infty} \left[(r_h - c)\min\left(xH(\hat{v}_p(r_s^p)), k\right) + (r_s^p - c)\min\left(x\bar{H}\left(\hat{v}_p\left(r_s^t\right)\right), k\right)\right] \mathrm{d}G(x) - 2c_F <$$

$$< \int_0^{x^\circ} \left[(r_h - c)\min(x, k) - c_F\right] \mathrm{d}G(x) + \int_{x^\circ}^{+\infty} \left[(r_h - c)\min(xH(\hat{v}_p(r_s^p)), k) + (r_s^p - c)\min\left(x\bar{H}\left(\hat{v}_p\left(r_s^t\right)\right), k\right) - 2c_F\right] \mathrm{d}G(x) <$$

$$< \int_0^{x^\circ} \left[(r_h - c)\min(x, k) - c_F\right] \mathrm{d}G(x) + \int_{x^\circ}^{+\infty} \left[(r_h - c)\min(xH(\hat{v}_t(r_s^p, n)), k) + (r_s^p - c)\min\left(x\bar{H}(\hat{v}_t(r_s^p, n)), k\right) - 2c_F\right] \mathrm{d}G(x) =$$

$$= \Pi_t(r_s^p, n) \leq \Pi_t\left(r_s^t, n^t\right),$$

where the first inequality follows from the definition of x°, the second because $H(\hat{v}_t(r_s^p, n)) \in \left[\frac{1}{2}, H(\hat{v}_p(r_s^p))\right]$ implies higher expected sales and higher average margin for all market realizations equal or higher than x°, and where $n = x^\circ \bar{H}\left(\hat{v}_t(r_s^p, n)\right)$ always admits a solution, as shown in Lemma 8 below.

If instead $\Pi_p < \Pi_c$, it is easy to check that $\Pi_t(r_s^p, n) > \Pi_c$. The result is then proven because $\Pi_a < \Pi_t(r_s^p, n) \leq \Pi_t$.

B.8. **Additional Lemma:**

Lemma 8. *Under Threshold Discounting, for any relevant price choice $r_s \geq \bar{r}$, the service provider can always choose the activation threshold n so that the deal is active when the market size is higher than a desired level m. Further, such activation threshold is unique. That is, $\forall r_s \in (\bar{r}, r_h)$ and $\forall m > 0\, \exists! n\left(r_s, m\right)$: $n\left(r_s, m\right)/\bar{H}\left(\hat{v}_t\left(r_s, n\left(r_s, m\right)\right)\right) = m$.*

Note that $n/\bar{H}\left(\hat{v}_t\left(r_s, n\right)\right)$ is continuous in n. Also, note that $\lim_{n\to 0} n/\bar{H}\left(\hat{v}_t\left(r_s, n\right)\right) = 0$ and further note that $\lim_{n\to+\infty} n/\bar{H}\left(\hat{v}_t\left(r_s, n\right)\right) = +\infty$. For the former, note that $\bar{H}\left(\hat{v}_p\right) > 0$, as it can be seen by looking at (B.9) and noting that for any $r \in [\bar{r}, r_h]$ we have $H\left(\hat{v}_p\left(r\right)\right) > 1/2$, hence $RHS < 1$, implying $\hat{v}_p\left(r\right) < v_h - (r_h - r) < v_h$, hence $\lim_{n\to 0} \bar{H}\left(\hat{v}_t\left(r_s, n\right)\right) = \bar{H}\left(\hat{v}_p\left(r_s\right)\right) > 0$. It is then left to show that $\frac{d}{dn}\left(n/\bar{H}\left(\hat{v}_t\left(r_s, n\right)\right)\right) > 0$. Suppose not. Then, it must be that $\frac{d}{dn}\bar{H}\left(\hat{v}_t\left(r_s, n\right)\right) > 0$, which implies $\frac{\partial}{\partial n}\hat{v}_t\left(r_s, n\right) < 0$ and hence $\frac{d}{dn} LHS_t\left(\hat{v}_t\left(r_s, n\right), r_s\right) < 0$.

From $\frac{d}{dy}\left(\int_y^{+\infty} \min\left(1, k/(H\left(\hat{v}_t\right) x)\right) \mathrm{d}G_c(x) \,/\, \int_y^{+\infty} \min\left(1, k/(\bar{H}\left(\hat{v}_t\right) x)\right) \mathrm{d}G_c(x)\right) < 0$ and considering that $\frac{d}{d\hat{v}_t}\left(\min(1, k/(H\left(\hat{v}_t\right) x)) \,/\, \min\left(1, k/(\bar{H}\left(\hat{v}_t\right) x)\right)\right) < 0$, we have that $\frac{d}{dn}\left(n/\bar{H}\left(\hat{v}_t\left(r_s, n\right)\right)\right) \leq 0$, implying that $\frac{d}{dn} RHS_t(\hat{v}_t(r_s, n), n) > 0$, and $\frac{d}{dn} LHS_t\left(\hat{v}_t\left(r_s, n\right), r_s\right) < 0$ cannot hold.

B.8.1. *Reverse threshold discounting.* In order to isolate the effect of a responsive discounting decision on the firm profit, we consider a population of non-strategic customers. Let reverse threshold discounting be an offer in which the firm opens up in the slow period offering a discounted price $r_s < r_h$ if *less* than n customers subscribe to the offer. Then we have the following result–the result holds even when $c_F = 0$.

Lemma 9. *Suppose that customers are non-strategic. Then, reverse threshold discounting yields a lower profit compared to the traditional approaches.*

Proof. Let $(r_s^r, n^r) = \arg\max_{r_s,n} \Pi_r^\mu (r_s, n)$, where

$$\Pi_r^\mu (r_s, n) = \int_0^{n/\alpha_s(r_s;\mu)} \left[r_h \min(k, \alpha_h (r_s;\mu) x) + r_s \min(k, \alpha_s (r_s;\mu) x) - 2c_F\right] \mathrm{d}G(x)$$

$$+ \int_{n/\alpha_s(r_s;\mu)}^{+\infty} \left[r_h \min(k, x) - c_F\right] \mathrm{d}G(x),$$

with $\alpha_h (r_s;\mu) = H(r_s + v_h - r_h)$ and $\alpha_s (r_s;\mu) = \bar{H}(r_s + v_h - r_h)$. We then have two cases. If $n^r/\alpha_s (r_s^r;\mu) < x^\circ$, then

$$\Pi_r = \int_0^{n^r/\alpha_s(r_s^r;\mu)} \left[r_h \min(k, \alpha_h (r_s^r;\mu) x) + r_s^r \min(k, \alpha_s (r_s^r;\mu) x) - 2c_F\right] \mathrm{d}G(x) +$$

$$+ \int_{n^r/\alpha_s(r_s^r;\mu)}^{+\infty} \left[r_h \min(k, x) - c_F\right] \mathrm{d}G(x) <$$

$$< \int_0^{n^r/\alpha_s(r_s;\mu)} \left[r_h \min(k, x) - c_F\right] \mathrm{d}G(x) + \int_{n^r/\alpha_s(r_s;\mu)}^{+\infty} \left[r_h \min(k, x) - c_F\right] \mathrm{d}G(x) = \Pi_c;$$

if instead $n^r/\alpha_s (r_s;\mu) > x^\circ$, then

$$\Pi_r = \int_0^{n^r/\alpha_s(r_s^r;\mu)} \left[r_h \min(k, \alpha_h (r_s^r;\mu) x) + r_s^r \min(k, \alpha_s (r_s^r;\mu) x) - 2c_F\right] \mathrm{d}G(x) +$$

$$+ \int_{n^r/\alpha_s(r_s^r;\mu)}^{+\infty} \left[r_h \min(k, x) - c_F\right] \mathrm{d}G(x) <$$

$$< \int_{0}^{n^r/\alpha_s(r_s;\mu)} \left[r_h \min\left(k, \alpha_h\left(r_s^r;\mu\right)x\right) + r_s^r \min\left(k, \alpha_s\left(r_s^r;\mu\right)x\right) - 2c_F\right] \mathrm{d}G\left(x\right) +$$

$$+ \int_{n^r/\alpha_s(r_s;\mu)}^{+\infty} \left[r_h \min\left(k, \alpha_h\left(r_s^r;\mu\right)x\right) + r_s^r \min\left(k, \alpha_s\left(r_s^r;\mu\right)x\right) - 2c_F\right] \mathrm{d}G\left(x\right) =$$

$$= \Pi_p^{\mu}\left(r_s^r\right) \leq \Pi_p^{\mu}.$$

□

B.9. **The Strategic scarcity effect.** See subsubsection B.7.1

B.10. **Theorem 8.** Suppose a fraction $\gamma \in (0,1)$ of the population is strategic, the rest being myopic, i.e. they do not account for the strategy of other customers, hence their visit decision is only a function of price, where a customer with valuation for the slow period v_s is better off subscribing, and visiting in the slow period conditional on the deal being active, iff $v_s - r_s > v_h - r_h$. Define $\hat{v}_m\left(r_s\right) = v_h - r_h + r_s$ as the valuation of the myopic indifferent customer, and let

$$\alpha_t\left(r_s, n, \gamma\right) = \left(1-\gamma\right) H\left(\hat{v}_m\left(r_s\right)\right) + \gamma H\left(\hat{v}_t\left(r_s, n, \gamma\right)\right)$$

be the fraction of customers subscribing to the deal (and visiting in the slow period when the deal is on), where $\hat{v}_t\left(r_s, n, \gamma\right)$ is the solution to

$$\text{(B.10)} \qquad \frac{\hat{v}_t - r_s}{v_h - r_h} = \frac{\int_{\frac{n}{(1-\gamma)H(\hat{v}_m(r_s))+\gamma H(\hat{v}_t)}}^{+\infty} \min\left(1, \frac{k}{x((1-\gamma)H(\hat{v}_m(r_s))+\gamma H(\hat{v}_t))}\right) \mathrm{d}G_c(x)}{\int_{\frac{n}{(1-\gamma)H(\hat{v}_m(r_s))+\gamma H(\hat{v}_t)}}^{+\infty} \min\left(1, \frac{k}{x\left((1-\gamma)\bar{H}(\hat{v}_m(r_s))+\gamma \bar{H}(\hat{v}_t)\right)}\right) \mathrm{d}G_c(x)}$$

and is unique in equilibrium since the LHS is increasing in $\hat{v}_t$ and the RHS is decreasing in $\hat{v}_t$ when $r_s : \left(\left(1-\gamma\right) H\left(\hat{v}_m\left(r_s\right)\right) + \gamma H\left(\hat{v}_t\right)\right) \geq \frac{1}{2}$, which can be shown to be always true in equilibrium following the same steps as for the case $\gamma = 1$. It is easy to prove that all monotonicity properties needed to prove the results in the previous sections are maintained, the only non-obvious being $\frac{d}{dr_s}\alpha_t\left(r_s, n, \gamma\right) > 0$, i.e. that an increase in price shifts more demand to the hot period, mainly because now both the LHS and RHS of (B.10) are a function of the price r_s, hence the old arguments are no longer sufficient. To show that $\frac{d}{dr_s}\alpha_t\left(r_s, n, \gamma\right) > 0$, let $LHS_t\left(\hat{v}_t\left(r_s, n, \gamma\right), r_s\right)$ and $RHS_t\left(\hat{v}_t\left(r_s, n, \gamma\right), \hat{v}_m\left(r_s\right), \gamma, n\right)$ be the LHS and RHS of (B.10). It is trivial to show that $\alpha_t\left(r_s^t, n^t, \gamma\right) \geq \frac{1}{2} \forall \gamma$. Now, suppose that $\frac{d}{dr_s}\alpha_t\left(r_s, n, \gamma\right) \leq 0$ for some r_s, n, γ. Then $0 < \frac{d}{dr_s}\left(1-\gamma\right) H\left(\hat{v}_m\left(r_s\right)\right) \leq -\frac{d}{dr_s}\gamma H\left(\hat{v}_t\left(r_s, n, \gamma\right)\right)$, hence $\frac{d}{dr_s}\hat{v}_t\left(r_s, n, \gamma\right) < 0$. Since $r_s \geq \bar{r} \Rightarrow \alpha_t\left(r_s, n, \gamma\right) \geq \frac{1}{2} \forall \gamma$ and $\frac{d}{dr_s}\alpha_t(r_s, n, \gamma) \leq 0$, we have $\frac{d}{dr_s}RHS_t(\hat{v}_t(r_s, n, \gamma), \hat{v}_m(r_s), \gamma, n) \geq 0$, hence it must be $\frac{d}{dr_s}LHS_t(\hat{v}_t(r_s,n,\gamma),r_s)\geq 0$, but this is impossible because $\frac{d}{dr_s}LHS_t\left(\hat{v}_t\left(r_s,n,\gamma\right),r_s\right) = (v_h-r_h)^{-1}(\frac{d}{dr_s}\hat{v}_t-\frac{d}{dr_s}r_s)<0$ since $\frac{d}{dr_s}\hat{v}_t\left(r_s,n,\gamma\right)<0$. The result is then proven by contradiction.

B.11. **Theorem 9.** First we show that, $\forall r_s\geq\bar{r}$ and $1>\gamma_2>\gamma_1>0$, we have that $\hat{v}_t\left(r_s,n,\gamma_2\right)>\hat{v}_t\left(r_s,n,\gamma_1\right)$. Define $\hat{\alpha}_t\left(r_s,\hat{v},\gamma\right)=\left(1-\gamma\right)H\left(\hat{v}_m\left(r_s\right)\right)+\gamma H\left(\hat{v}_t\right)$. Note that, $\forall r_s\geq\bar{r}$ and $1>\gamma_2>\gamma_1>0$, we have that $RHS_t\left(\hat{v}_t\left(r_s,n,\gamma_1\right),\hat{v}_m\left(r_s\right),\gamma_1,n\right)<RHS_t\left(\hat{v}_t\left(r_s,n,\gamma_1\right),\hat{v}_m\left(r_s\right),\gamma_2,n\right)$. In fact

$$RHS_t\left(\hat{v}_t\left(r_s,n,\gamma_1\right),\hat{v}_m\left(r_s\right),\gamma_1,n\right)=\frac{\int_{\frac{n}{1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_1\right)}}^{+\infty}\min\left(1,\frac{k}{x\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_1\right)}\right)\mathrm{d}G_c(x)}{\int_{\frac{n}{1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_1\right)}}^{+\infty}\min\left(1,\frac{k}{x(1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_1\right))}\right)\mathrm{d}G_c(x)}<$$

$$<\frac{\int_{\frac{n}{1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_1\right)}}^{+\infty}\min\left(1,\frac{k}{x\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_2\right)}\right)\mathrm{d}G_c(x)}{\int_{\frac{n}{1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_1\right)}}^{+\infty}\min\left(1,\frac{k}{x(1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_2\right))}\right)\mathrm{d}G_c(x)}\leq\frac{\int_{\frac{n}{1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_2\right)}}^{+\infty}\min\left(1,\frac{k}{x\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_2\right)}\right)\mathrm{d}G_c(x)}{\int_{\frac{n}{1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_2\right)}}^{+\infty}\min\left(1,\frac{k}{x(1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma_2\right))}\right)\mathrm{d}G_c(x)},$$

where the last term is equal to $RHS_t\left(\hat{v}_t\left(r_s,n,\gamma_1\right),\hat{v}_m\left(r_s\right),\gamma_2,n\right)$. The first inequality comes by noting that $\min\left(1,k/(x\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma\right))\right)/\min\left(1,k/(x\left(1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma\right)\right))\right)$ is decreasing in γ due to $\frac{\partial}{\partial\gamma}\hat{\alpha}_t\left(r_s,\hat{v}_{t,\gamma_1},\gamma\right)>0$ since $\hat{v}_{t,\gamma_1}<v_h-r_h+r_s$, which comes from $(\hat{v}_t\left(r_s,n,\gamma_1\right)-r_s)/v_h-r_h=LHS_t\left(\hat{v}_t\left(r_s,n,\gamma_1\right),r_s\right)$ which is also equal to $RHS_t\left(\hat{v}_t\left(r_s,n,\gamma_1\right),\hat{v}_m\left(r_s\right),\gamma_1,n\right)$, which is less than one, hence less than $(\hat{v}_m\left(r_s\right)-r_s)/(v_h-r_h)$, implying that $\hat{v}_m\left(r_s\right)>\hat{v}_t\left(r_s,n,\gamma_1\right)$. The second inequality comes from $\frac{\partial}{\partial\gamma}\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma\right)>0$ and from $\min\left(1,k/(x\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma\right)))\right)/\min\left(1,k/(x\left(1-\hat{\alpha}_t\left(r_s,\hat{v}_t,\gamma\right)\right)))\right)$ decreasing in x. $RHS_t(\hat{v}_t(r_s,n,\gamma_1),\hat{v}_m\left(r_s\right),\gamma_1,n)<RHS_t\left(\hat{v}_t\left(r_s,n,\gamma_1\right),\hat{v}_m\left(r_s\right),\gamma_2,n\right)$, and since $LHS_t(\hat{v}_t,r_s)$ increases but $RHS_t(\hat{v}_t\left(r_s,n,\gamma\right),\hat{v}_m\left(r_s\right),\gamma,n)$ decreases in $\hat{v}_t$, then for the condition $LHS_t(\hat{v}_t(r_s,n,\gamma_2),r_s)=RHS_t\left(\hat{v}_t\left(r_s,n,\gamma_2\right),\hat{v}_m\left(r_s\right),\gamma_2,n\right)$ to hold it must be that $\hat{v}_t\left(r_s,n,\gamma_2\right)>\hat{v}_t\left(r_s,n,\gamma_1\right)$.

If we now rewrite $RHS_t\left(\hat{v}_t\left(r_s,n,\gamma\right),\hat{v}_m\left(r_s\right),\gamma,n\right)$ as $RHS_t\left(\alpha_t\left(r_s,n,\gamma\right),n\right)$, it must be that

$$RHS_t\left(\alpha_t\left(r_s,n,\gamma_1\right)\right)=LHS_t\left(\hat{v}_t\left(r_s,n,\gamma_1\right),r_s\right)<LHS_t\left(\hat{v}_t\left(r_s,n,\gamma_2\right),r_s\right)=RHS_t\left(\alpha_t\left(r_s,n,\gamma_2\right)\right),$$

since $RHS_t\left(\alpha_t\left(r_s,n,\gamma\right),n\right)$ decreases in α_t, it follows that $\alpha_t\left(r_s,n,\gamma_1\right)>\alpha_t\left(r_s,n,\gamma_2\right)$. The result is then proven following the usual argument that since demand is more evenly balanced between the two periods with γ_2 compared to γ_1, the service provider can always earn a higher profit by increasing the price in the slow period so to achieve in every market state the same capacity utilization and a strictly higher average margin.

B.12. **Type II equilibria do not exist when there are myopic customers in the population.** When a fraction γ of customers in the population are strategic, the incremental expected surplus Δu for customer i with slow-period valuation $v_{s,i}$ by subscribing to the deal when all other strategic customers do not subscribe is

$$\Delta u\left(v_{s,i};\hat{v}_{t,-i}=v_h,\gamma\right)=\bar{G}_c\left(n_t\bar{\alpha}_m^{-1}\right)\int_{n_t\bar{\alpha}_m^{-1}}^{+\infty}\left[\min\left(1,k\,x^{-1}\right)\left(v_h-r_h\right)-\min\left(1,k\left(\bar{\alpha}_m x\right)^{-1}\right)\left(v_s-r_s\right)\right]\mathrm{d}G_c\left(x\right),$$

where $\bar{\alpha}_m=(1-\gamma)\,\bar{H}\left(v_h-r_h+r_s\right)$ is the fraction of the population that subscribes to the deal and visits in the slow period when the deal is active. For $v_{s,i}$ high enough and $\gamma<1$, customer i has a strictly positive incremental surplus from subscribing, hence there can exists no equilibrium where strategic customers coordinate to never subscribe to the deal.

Design considerations in Threshold Discounting Offers.

B.13. **Theorem 10.** In order to show that committing to a threshold activation strategy does not reduce the ability of the service provider to exploit the information that the number of subscribers reveals, we need to show that the optimal activation decision is always of a threshold type. First, note that for any activation strategy $\mathcal{A}\left(r_o,n\right)$ that the firm may use after announcing a discounted price r_o and upon observing the number of subscribers n, $\mathcal{A}\left(r_o,n\right):[0,r_h]\times\mathbb{R}_+\longmapsto\{0,1\}$, where 1 stands for activate and 0 stands for not activate, the optimal subscription strategy for customers is always of a threshold type, since (B.3) still applies, as the expected gain from subscribing always increases in customer valuation for the slow period v_s. This implies that a higher number of subscribers is associated to a higher market realization. Note also that the service provider can always infer market realization x upon observing the number of subscribers n once he knows customer subscription strategy $\hat{v}_o$.

Consider now the expected profit gain of the service provider from activating the deal for a given market realization x and an announced price r_o:

$$\min\left(k,xH\left(\hat{v}_o\right)\right)\left(r_h-c\right)+\min\left(k,x\bar{H}\left(\hat{v}_o\right)\right)\left(r_o-c\right)-c_F-\min\left(k,x\right)\left(r_h-c\right).$$

Similarly to what shown in Lemma 2, there exists a market realization $x_o^{\circ}\left(r_o\right)$ such that the above is negative for $x\leq x_o^{\circ}\left(r_o\right)$ and positive otherwise, with $x_o^{\circ}\left(r_o\right)$ being the market realization that makes the provider indifferent between activating or not. It follows that the optimal activation decision is always to activate the deal iff $x>x_o^{\circ}\left(r_o\right)$, hence the provider can set this decision upfront without any loss in profit. On the other hand, when the activation threshold is announced upfront, customer subscription decision is a function of it. Let $\Pi_o\left(r_s^o\right)$ be the equilibrium profit under Opaque activation rule. Then we have

$$\Pi_o\left(r_s^o\right)=\int_0^{x_o^{\circ}\left(r_s^o\right)}\left(\min(k,x)(r_h-c)-c_F\right)\mathrm{d}G(x)+$$

$$+\int_{x_o^\circ(r_s^o)}^{+\infty}\left(\min(k, x H(\hat{v}_o(r_s^o)))(r_h-c)+\min\left(k, x\bar{H}(\hat{v}_o(r_s^o))\right)(r_s^o-c)-2c_F\right)\mathrm{d}G(x) =$$

$$= \int_0^{\frac{n^\circ}{\bar{H}(\hat{v}_t(r_s^o,n^\circ))}}(\min(k,x)(r_h-c)-c_F)\mathrm{d}G(x) +$$

$$+\int_{\frac{n^\circ}{\bar{H}(\hat{v}_t(r_s^o,n^\circ))}}^{+\infty}\left(\min\left(k, x H\left(\hat{v}_t\left(r_s^o, n^\circ\right)\right)\right)(r_h-c)+\min\left(k, x\bar{H}\left(\hat{v}_t\left(r_s^o, n^\circ\right)\right)\right)\left(r_s^o-c\right)-2c_F\right)\mathrm{d}G(x) =$$

$$= \Pi_t\left(r_s^o, n^\circ\right) \leq \Pi_t\left(r_s^t, n^t\right)$$

where the second inequality comes from the fact that for every announced price r_s^o under Threshold Discounting, the provider can always set a threshold n° such that $n^\circ/\bar{H}\left(\hat{v}_t\left(r_s^o, n^\circ\right)\right) = x_o^\circ\left(r_s^o\right)$, for Lemma 8.

B.14. **Theorem 11.** The equilibrium for a threshold discounting policy with late disclosure is very similar to the one arising from threshold discounting and so is the proof, which is omitted for brevity. The customer subscription and visit strategy $\hat{v}_l\left(r_s, n\right)$ is given by

$$\frac{\hat{v}_l - r_s}{v_h - r_h} = \frac{\int_0^{+\infty}\min\left(1, \frac{k}{H(\hat{v}_l)x}\right)\mathrm{d}G_c(x)}{\int_{\frac{n}{\bar{H}(\hat{v}_t)}}^{+\infty}\min\left(1, \frac{k}{\bar{H}(\hat{v}_l)x}\right)\mathrm{d}G_c(x)}, \tag{B.11}$$

and the profit is given by

$$\Pi_l = \max_{r_s,n}(r_h-c)\int_0^{\frac{n}{\bar{\alpha}_l(r_s,n)}}\left[\min(k,\alpha_l(r_s,n)x)-c_F\right]\,\mathrm{d}G(x)+$$

$$+\int_{\frac{n}{\bar{\alpha}_l(r_s,n)}}^{+\infty}\left[\min(k,\alpha_l(r_s,n)x)(r_h-c)+\min(k,\bar{\alpha}_t(r_s,n)x)(r_s-c)-2c_F\right]\,\mathrm{d}G(x)$$

$$s.t.\, r_s < r_h,\ n > 0,$$

where $\alpha_l\left(r_s,n\right) = H\left(\hat{v}_l\left(r_s,n_t\right)\right)$ is the fraction of customers subscribing to the deal, and where $\bar{\alpha}_t\left(r_s,n\right) = 1-\alpha_t\left(r_s,n\right)$. Let's define the LHS and RHS of (B.11) as $LHS_l\left(\hat{v}_l, r_s\right)$ and $RHS_l\left(\hat{v}_l, n\right)$. Let $\hat{n}_t\left(r\right)$: $\hat{n}_t/(1-\alpha_t\left(r,\hat{n}_t\right)) = n/\bar{\alpha}_l(r_s,n)$, and take $\hat{r}_t \geq r_s :\ \alpha_t\left(\hat{r}_t,\hat{n}_t\right) \geq \frac{1}{2}$, which clearly always exists. Then we have that

$$\Pi_l = (r_h-c)\int_0^{\frac{n}{\bar{\alpha}_l(r_s^l,n^l)}}\left[\min\left(k,\alpha_l\left(r_s^l,n^l\right)x\right)-c_F\right]\mathrm{d}G(x)+\int_{\frac{n^l}{\bar{\alpha}_l(r_s^l,n^l)}}^{+\infty}\left[\min\left(k,\alpha_l\left(r_s^l,n^l\right)x\right)(r_h-c)+\min\left(k,\bar{\alpha}_t\left(r_s^l,n^l\right)x\right)\left(r_s^l-c\right)-2c_F\right]\mathrm{d}G(x) <$$

$$< (r_h - c)\int_0^{\frac{n^l}{\bar\alpha_l(r_s^l,n^l)}} [\min(k,x) - c_F]\ \mathrm{d}G(x) + \int_{\frac{n^l}{\bar\alpha_l(r_s^l,n^l)}}^{+\infty} \left[\min\left(k,\alpha_l\left(r_s^l,n^l\right)x\right)(r_h - c) + \min\left(k,\bar\alpha_t\left(r_s^l,n^l\right)x\right)\left(r_s^l - c\right) - 2c_F\right]\ \mathrm{d}G(x) \leq$$

$$\leq (r_h - c)\int_0^{\frac{\hat n_t}{1-\alpha_t(\hat r_t,\hat n_t)}} (\min(k,x) - c_F)\mathrm{d}G(x) + \int_{\frac{\hat n_t}{1-\alpha_t(\hat r_t,\hat n_t)}}^{+\infty} (\min(k,\alpha_t(\hat r_t,\hat n_t)x)(r_h - c) + \min(k,(1-\alpha_t(\hat r_t,\hat n_t))x)\left(r_s^l - c\right) - 2c_F)\mathrm{d}G(x) =$$

$$= \Pi_t(\hat r_t,\hat n_t) \leq \Pi_t.$$

B.15. **Theorem 12.** With arguments similar to the ones used for Threshold Discounting, it can be shown that there exists a unique $\hat v_u(r_u,n_u)$ such that customers do not subscribe and always consume in the hot period if $v_s \leq \hat v_u(r_u,n_u)$, and subscribe to the deal and visit in the slow period iff the deal is on otherwise, with $\hat v_u$ being given by

$$\frac{\hat v_u - r}{v_h - r} = \frac{\int_{\frac{n_u}{H(\hat v_u)}}^{+\infty} \min\left(1, \frac{k}{H(\hat v_u)x}\right)\mathrm{d}G_c(x)}{\int_{\frac{n_u}{\bar H(\hat v_h)}}^{+\infty} \min\left(1, \frac{k}{H(\hat v_u)x}\right)\mathrm{d}G_c(x)}.$$

Let $LHS_u(\hat v_u, r_u)$ and $RHS_u(\hat v_u, n_u)$ be the LHS and RHS of the above equation. For any price r_u and threshold n_u, we have $H(\hat v_u(r_u,n_u)) > 1/2$ as the discount is offered on both periods, making the hot period always the more attractive regardless of the pricing decision. Clearly, $RHS_t(\hat v, n) = RHS_u(\hat v, n)$ and $LHS_t(\hat v, r) > LHS_u(\hat v, r)$. It follows that under Threshold Discounting the provider achieves a higher profit by choosing $r_s' > r_s^u$ and $n_t'(r_s')$ such that $n_t'\bar H(\hat v_t(r_s', n_t'))^{-1} = n^u \bar H(\hat v_u(r_s^u, n^u))^{-1}$, which is always possible in light of Lemma 8, and so that both $r_s' \leq r_h$ and $H(\hat v_t(r_s', n_t')) \leq H(\hat v_u(r_s^u, n^u))$ are satisfied with at least one of them binding, which is accomplished setting r_s' high enough, due to LHS_t and $H(\hat v_t)$ being increasing with respect to r_s. Formally, letting $\alpha_u^* = H(\hat v_u(r_s^u, n^u))$ and $\alpha_t' = H(\hat v_t(r_s', n_t'))$, we have

$$\Pi_u^* = \int_0^{\frac{n^u}{1-\alpha_u^*}} [\min(k,x)(r_h - c) - c_F]\,\mathrm{d}G(x) + \int_{\frac{n^u}{1-\alpha_u^*}}^{+\infty} [\min(k,\alpha_u^* x)(r_s^u - c) + \min(k,(1-\alpha_u^*)x)(r_s^u - c) - 2c_F]\,\mathrm{d}G(x) <$$

$$< \int_0^{\frac{n^u}{1-\alpha_u^*}} [\min(k,x)(r_h - c) - c_F]\,\mathrm{d}G(x) + \int_{\frac{n^u}{1-\alpha_u^*}}^{+\infty} \left[\min\left(k,\alpha_t' x\right)(r_h - c) + \min\left(k,\left(1-\alpha_t'\right)x\right)(r_s' - c) - 2c_F\right]\mathrm{d}G(x) =$$

$$= \Pi_t(r_s', n_t') \leq \Pi_t,$$

where the inequality follows noting that $\min(k,\alpha_u^* x)(r_s^u - c) + \min(k,(1-\alpha_u^*)x)(r_s^u - c)$ is strictly less than $\min\left(k,\alpha_t' x\right)(r_h - c) + \min\left(k,\left(1-\alpha_t'\right)x\right)(r_s' - c)$ for every $x \geq \frac{n^u}{1-\alpha_u^*}$.

B.16. **Theorem 13.** Suppose, conservatively, that the firm purchases the extra service included in the offer from an external provider at the same cost that a customer would, i.e. without volume discounts or commissions for the advertisement and extra demand provided to the external provider–otherwise, this alone would already make focused threshold discounting better than classic threshold discounting. Using the same arguments as before, one can show that the subscription strategy is a threshold strategy, and that the visit strategy is as well. For any promotion ζ and activation threshold n_f, let $\hat{v}_s$ be the valuation of a customer that, conditional on the deal being active, is indifferent between the two periods given that she is provided no additional incentives, and let also $\hat{v}_f$ be the valuation of a customer that, conditional on the deal being active, is indifferent between the two periods given that she earns (or is refunded) a service worth ζV, in case she visits in the slow period. Clearly, $\hat{v}_s \geq \hat{v}_f$. Since people that find no value in the external service do not subscribe, let $\sigma_f(\hat{v}_f) = H(\bar{v}_s) - H(\hat{v}_f)$ be the fraction of the market that subscribes, who will also visit when deal is on. In addition to them, some non-subscribers will also visit: let $\rho_f(\hat{v}_s) = \bar{H}(\max(\hat{v}_s, \bar{v}_s))$ be the fraction of the market that visits when the deal is on even without additional incentives. Since a customer subscribes only if she is better off visiting when the deal is on, in equilibrium we must have

$$\text{(B.12)} \qquad \frac{\hat{v}_s - r_h}{v_h - r_h} = \frac{\int_{n_f\sigma_f(\hat{v}_f)^{-1}}^{+\infty} \min\left(1, k\left(\alpha_f(\hat{v}_s, \hat{v}_f)\, x\right)^{-1}\right) \mathrm{d}G_c(x)}{\int_{n_f\sigma_f(\hat{v}_f)^{-1}}^{+\infty} \min\left(1, k\left(\bar{\alpha}_f(\hat{v}_s, \hat{v}_f)\, x\right)^{-1}\right) \mathrm{d}G_c(x)} = \frac{\hat{v}_f - r_h + \zeta V}{v_h - r_h},$$

where $\alpha_f(\hat{v}_s, \hat{v}_f) = \sigma_f(\hat{v}_f) + \rho_f(\hat{v}_s)$ is the fraction of the market visiting in the slow period when the deal is active, and $\bar{\alpha}_f(\hat{v}_s, \hat{v}_f) = 1 - \alpha_f(\hat{v}_s, \hat{v}_f)$.

We now show that there exists a threshold n'_f and a coupon ζ' such that $\alpha_f\left(\hat{v}'_s, \hat{v}'_f\right) = \alpha_t(r_s^t, n^t)$ and also $n'_f\sigma_f\left(\hat{v}'_f\right)^{-1} = n^t\alpha_t(r_s^t, n^t)^{-1}$, so that

$$\Pi_t = (r_h - c)\int_0^{n^t\bar{\alpha}_t^{*-1}} \min(k, x) - c_F \,\mathrm{d}G(x) + \int_{n^t\bar{\alpha}_t^{*-1}}^{+\infty} \min(k, \alpha_t^* x)(r_h - c) + \min(k, \bar{\alpha}_t^* x)\left(r_s^t - c\right) - 2c_F \,\mathrm{d}G(x) <$$

$$< (r_h - c)\int_0^{n'_f\bar{\alpha}'^{-1}_f} \min(k, x) - c_F \,\mathrm{d}G(x) + \int_{n'_f\bar{\alpha}'^{-1}_f}^{+\infty} \min\left(k, \alpha'_f x\right)(r_h - c) + \min\left(k, \bar{\alpha}'_f x\right)\left(r_h - c - \frac{\sigma'_f}{\alpha'_f}\zeta' V\right) - 2c_F \,\mathrm{d}G(x) =$$

$$= \Pi_f\left(\zeta', n'_f\right) < \Pi_f,$$

where Π_f is the equilibrium profit under focused threshold discounting. Take the announcement $n'_f = n^t(H(\bar{v}_s) - H(\hat{v}_t^*))/\bar{H}(\hat{v}_t^*)$, $\zeta' = V^{-1}(r_h - r_s^t)$. Then (B.12) is satisfied for $\hat{v}'_f = \hat{v}_t^*$ and $\hat{v}_s = \breve{v}_t(r_s^t, n^t)$, where $\breve{v}_t(r_s^t, n^t)$ is the valuation of the customer that under threshold discounting, in equilibrium, would get the same expected surplus in both periods when charged full price on both. In fact,

$$\frac{\int_{n'_f\sigma_f(\hat{v}'_f)^{-1}}^{+\infty} \min\left(1, k\left(\alpha_f\left(\hat{v}'_s, \hat{v}'_f\right) x\right)^{-1}\right) \mathrm{d}G_c(x)}{\int_{n'_f\sigma_f(\hat{v}'_f)^{-1}}^{+\infty} \min\left(1, k\left(\bar{\alpha}_f\left(\hat{v}'_s, \hat{v}'_f\right) x\right)^{-1}\right) \mathrm{d}G_c(x)} = \frac{\int_{n^t\bar{\alpha}_t(r_s^t, n^t)^{-1}}^{+\infty} \min\left(1, k\left(\alpha_t(r_s^t, n^t)\, x\right)^{-1}\right) \mathrm{d}G_c(x)}{\int_{n^t\bar{\alpha}_t(r_s^t, n^t)^{-1}}^{+\infty} \min\left(1, k\left(\bar{\alpha}_t(r_s^t, n^t)\, x\right)^{-1}\right) \mathrm{d}G_c(x)}$$

$$\frac{\hat{v}_s'-r_h}{v_h-r_h}=\frac{\breve{v}_t(r_s^t,n^t)-r_h}{v_h-r_h}=\frac{\int_{n^t\bar{\alpha}_t(r_s^t,n^t)^{-1}}^{+\infty}\min\Big(1,k\left(\alpha_t\left(r_s^t,n^t\right)x\right)^{-1}\Big)\,\mathrm{d}G_c(x)}{\int_{n^t\bar{\alpha}_t(r_s^t,n^t)^{-1}}^{+\infty}\min\Big(1,k\left(\bar{\alpha}_t\left(r_s^t,n^t\right)x\right)^{-1}\Big)\,\mathrm{d}G_c(x)}=\frac{\hat{v}_t(r_s^t,n^t)-r_s^t}{v_h-r_h}=\frac{\hat{v}_f'-r_h+\zeta'V}{v_h-r_h}.$$

We are left to show that $\left(\hat{v}_s',\hat{v}_f'\right)$ is the unique solution to (B.12). To keep notation simple, let the three members of (B.12) be renamed as $LHS_s\left(\hat{v}_s\right)$, $RHS_f\left(\hat{v}_s,\hat{v}_f,n_f\right)$, and $LHS_f\left(\hat{v}_F,\zeta\right)$. Note that $LHS_s\left(\hat{v}_s\right)$ and $RHS_f\left(\hat{v}_s,\hat{v}_f,n_f\right)$ are respectively increasing and decreasing in $\hat{v}_s$, that $LHS_f\left(\hat{v}_f,\zeta\right)$ increases in $\hat{v}_f$ and ζ, and that if $\alpha_f\left(\hat{v}_s,\hat{v}_f\right)>1/2$, we also have that $RHS_f\left(\hat{v}_s,\hat{v}_f,n_f\right)$ is decreasing in both $\hat{v}_f$ and n_f. Let's consider for now only the couples $\left(\hat{v}_s,\hat{v}_f\right):\ \alpha_f\left(\hat{v}_s,\hat{v}_f\right)\geq 1/2$. Clearly there can be only a unique solution to (B.12). In fact, suppose not, e.g. $\hat{v}_s>\bar{v}_s\geq\hat{v}_s'$, then the first equality of (B.12) requires $\hat{v}_f$ to decrease, and the second equality requires $\hat{v}_f$ to increase. The other case $\hat{v}_s<\bar{v}_s$ is trivial since σ_f does not change. Now suppose there exists a solution to (B.12) for $\left(\hat{v}_s^\bullet,\hat{v}_f^\bullet\right):\ \alpha_f\left(\hat{v}_s^\bullet,\hat{v}_f^\bullet\right)<1/2$. Then $RHS_f\left(\hat{v}_s^\bullet,\hat{v}_f^\bullet,n_f'\right)>1$, which implies $\hat{v}_f^\bullet<\hat{v}_f'$ and $\hat{v}_s^\bullet<\hat{v}_s'$, contradicting $\alpha_f\left(\hat{v}_s^\bullet,\hat{v}_f^\bullet\right)<1/2$ since $\alpha_f\left(\hat{v}_s,\hat{v}_f\right)$ decreases in both $\hat{v}_s$ and $\hat{v}_f$.

Threshold discounting.

C.1. **Equilibrium outcome.**

Equilibrium for the customer continuation game $\Gamma(r_s, n)$. The strategy of an individual customer i with valuation vector $\mathbf{v} = (v_h, v_s)$ is a vector $\mathcal{S}_i = \left(\sigma_i, \nu_i^{on}, \nu_i^{off}\right)$ where σ_i, ν_i^{on}, and ν_i^{off} are functions that, for every firm announcement (r_s, n), specify whether the customer subscribes $(\sigma_i(r_s, n, \mathbf{v}) = s)$ or not $(\sigma_i(r_s, n, \mathbf{v}) = ns)$, and conditional on the deal outcome $\omega \in \{on, off\}$, specify whether the customer visits on the hot period $(\nu_i^{\omega}(r_s, n, \mathbf{v}) = vh)$, in the slow period $(\nu_i^{\omega}(r_s, n, \mathbf{v}) = vs)$, or does not visit the firm $(\nu_i^{\omega}(r_s, n, \mathbf{v}) = v0)$. As an example, the vector (s, vs, vh) specifies that the customer subscribes, visits in the slow period when the deal is active, and visits in the hot period when the deal is not active. The strategy of each customers will be a function of the deal (r_s, n), but for the customer continuation game the deal terms are exogenous so we henceforth skip the dependance from (r_s, n) to avoid excess notation. We restrict to pure strategies for brevity, but the result of the analysis does not change if we include behavior strategies.

An equilibrium for the customer continuation game is characterized by the strategy profile $\mathcal{S} = \left(\sigma, \nu^{on}, \nu^{off}\right)$, where the absence of the subscript i means that $\sigma, \nu^{on}, \nu^{off}$ are vectors that summarize the strategies for every individual customer. An individual customer, after observing her valuation vector $\mathbf{v} = (v_h, v_s)$, has twelve possible pure strategies in the customer continuation game, given by the product $\{s, sn\} \times \{vs, vh, v0\} \times \{vh, v0\}$. Of these, six can be easily dismissed because either inconsistent or dominated by other strategies. For any given strategy profile $\mathcal{S}_{-i} = \left(\sigma_{-i}, \nu_{-i}^{on}, \nu_{-i}^{off}\right)$ of all customers except i, the strategy of customer i pairs each valuation vector (v_h, v_s) with one among six courses of action: (ns, vh, vh), (ns, vs, vh), $(ns, vs, v0)$, $(ns, v0, v0)$, (s, vs, vh), $(s, vs, v0)$.

Clearly $(ns, v0, v0)$ is optimal when $v_h < r_h$ and $v_s < r_s$, as all other strategies yield negative utility. (ns, vh, vh) is clearly optimal for $v_h > r_h$ and $v_s < r_s$, as all other strategies yield a non-positive utility, and $(s, vs, v0)$ is clearly optimal for $v_h < r_h$ and $v_s > r_s$ for the same reason. These strategies are equilibrium strategies for customer i regardless of $\mathcal{S}_{-i}$, hence are strategies followed by all customers in equilibrium. For a customer with $v_h > r_h$ and $v_s > r_s$, she will prefer (s, vs, vh) to (ns, vh, vh) if her slow period valuation is higher than a threshold level $\hat{v}_t(v_h)$, given by

$$(\hat{v}_t(v_h) - r_s) = (v_h - r_h)\frac{\int_{n/\alpha_s^t}^{+\infty}\min\left(1, k\left(\alpha_h^t x\right)^{-1}\right)\mathrm{d}G_c(x)}{\int_{n/\alpha_s^t}^{+\infty}\min\left(1, k\left(\alpha_s^t x\right)^{-1}\right)\mathrm{d}G_c(x)}, \tag{C.1}$$

where α_h^t and α_s^t are the fraction of the population visiting in the hot and in the slow period, according to $\mathcal{S}_{-i}$, and G_c refer to the updated market size distribution function upon customer existence in the market. Since all customers are ex-ante equal, in equilibrium they will all follow (C.1), with α_h^t and α_s^t representing the fraction of customers that do not subscribe and visit in the hot period, and that subscribe, respectively. The above equation shows a linear relation between v_s and v_h, hence the customers that are indifferent between not subscribing and visiting in the hot period, and subscribing and visiting in the slow period if the deal is active and in the hot period otherwise, have their valuation vector lay on a line with positive slope– the higher v_h, the higher the v_s needed to make a customer indifferent between (s, vs, vh) and (ns, vh, vh). This means that an IE will be characterized by four customer strategies, represented by the four areas separated by dotted borders in Figure 16.2, and that α_h^t and α_s^t are respectively given by $\alpha_h^t(r_s, n) = \int_{r_h}^{\bar{v}}\int_0^{r_s+(\hat{v}_t(\tau_h;r_s,n)-r_s)(\tau_h-r_h)(\bar{v}-r_h)^{-1}} h(\tau_h, \tau_s)\,\mathrm{d}\tau_s\mathrm{d}\tau_h$ and by the expression $\alpha_s^t(r_s, n) = \int_0^{\bar{v}}\int_{r_s+\left((\hat{v}_t(\tau_h;r_s,n)-r_s)(\tau_h-r_h)(\bar{v}-r_h)^{-1}\right)^+}^{\bar{v}} h(\tau_h, \tau_s)\,\mathrm{d}\tau_s\mathrm{d}\tau_h$. For the same structural properties of the ratio of integrals in the rhs of (C.1), the equilibrium is unique if the hot period is busier than the slow period. It is also easy to verify that this equilibrium Pareto dominates uninformative equilibria for the same reasons as under the base model (subsubsection B.4.6).

Firm profit maximizing deal. The firm then chooses the terms of the deal in order to maximize its expected profit, taking into account customer aggregate response, that is, $\left(r_s^t, n^t\right) = \arg\max_{r_s,n}\Pi_t(r_s, n)$ s.t. $r_s < r_h$ and $n > 0$, where $\Pi_t(r_s, n)$ is given by (16.3).

C.2. **Theorem 14.**

Proof. pick any $\hat{r} \in (c_F/k, r_h)$, and $\hat{n}$ as the solution to $\hat{n} = \max\left(k\frac{\alpha_s^t(\hat{r},\hat{n})}{\alpha_h^t(\hat{r},\hat{n})}, \frac{c_F}{\hat{r}}\right)$, where $\hat{n} < k$. Then, conditional on the deal being active, the hot period is full under threshold discounting. Clearly, $\alpha_h^c > \alpha_h^t(\hat{r}, \hat{n})$. Hence

$$\Pi_t(\hat{r},\hat{n}) = \int_0^{\hat{n}\alpha_s^t(\hat{r},\hat{n})^{-1}} [r_h\min(k, \alpha_h^c x) - c_F]\mathrm{d}G(x) + \int_{\hat{n}\alpha_s^t(\hat{r},\hat{n})^{-1}}^{+\infty}\left[r_h\min\left(k, \alpha_h^t(\hat{r},\hat{n})\,x\right) + \hat{r}\min\left(k, \alpha_s^t(\hat{r},\hat{n})\,x\right) - 2c_F\right]\mathrm{d}G(x) =$$

$$= \int_0^{\hat{n}\alpha_s^t(\hat{r},\hat{n})^{-1}} \left[r_h \min\left(k, \alpha_h^c x\right) - c_F\right] \mathrm{d}G\left(x\right) + \int_{\hat{n}\alpha_s^t(\hat{r},\hat{n})^{-1}}^{+\infty} \left[r_h k + \hat{r}\min\left(k, \alpha_s^t\left(\hat{r},\hat{n}\right)x\right) - 2c_F\right] \mathrm{d}G\left(x\right) =$$

$$= \int_0^{\hat{n}\alpha_s^t(\hat{r},\hat{n})^{-1}} \left[r_h \min\left(k, \alpha_h^c x\right) - c_F\right] \mathrm{d}G\left(x\right) + \int_{\hat{n}\alpha_s^t(\hat{r},\hat{n})^{-1}}^{+\infty} \left[r_h k - c_F + \hat{r}\min\left(k, \alpha_s^t\left(\hat{r},\hat{n}\right)x\right) - c_F\right] \mathrm{d}G\left(x\right) >$$

$$> \int_0^{\hat{n}\alpha_s^t(\hat{r},\hat{n})^{-1}} \left[r_h \min\left(k, \alpha_h^c x\right) - c_F\right] \mathrm{d}G\left(x\right) + \int_{\hat{n}\alpha_s^t(\hat{r},\hat{n})^{-1}}^{+\infty} \left[r_h k - c_F\right] \mathrm{d}G\left(x\right) = \Pi_c$$

where the last inequality follows from $\hat{r}\min\left(k, \alpha_s^t\left(\hat{r},\hat{n}\right)x\right) - c_F > 0$ for $x > \hat{n}\alpha_s^t\left(\hat{r},\hat{n}\right)^{-1}$, which follows since for $\hat{x} = \hat{n}\alpha_s^t\left(\hat{r},\hat{n}\right)^{-1}$ we have that $\hat{r}\min\left(k, \alpha_s^t\left(\hat{r},\hat{n}\right)\hat{x}\right) - c_F = \hat{r}\min\left(k, \alpha_s^t\left(\hat{r},\hat{n}\right)\hat{n}\alpha_s^t\left(\hat{r},\hat{n}\right)^{-1}\right) - c_F \geq \frac{c_F}{k}\min\left(k, \frac{c_F}{\hat{r}}\right) - c_F \geq 0$. □

C.3. **The three effects of discounting in threshold discounting.**

$$\frac{d}{d\theta}\Pi_p\left(\theta\right) = \underbrace{\int_0^{+\infty} \frac{d\alpha_h^p\left(\theta\right)}{d\theta} r_h \left(1_{x<k\left(\alpha_h^p(\theta)\right)^{-1}} - 1_{x<k\left(\alpha_s^p(\theta)\right)^{-1}}\left(1-\theta\right)\right) x \mathrm{d}G\left(x\right) +}_{operational\,(\pm)}$$

$$(C.2) \qquad + \Bigg[\underbrace{-\int_0^{+\infty} \min\left(k, \alpha_s^p\left(\theta\right)x\right) \mathrm{d}G\left(x\right)}_{margin\,(-)} + \underbrace{\int_0^{k\alpha_s^p(\theta)^{-1}} \left(r_h\left(1-\theta\right)\right) \frac{d\alpha_0^p\left(\theta\right)}{d\theta} x \mathrm{d}G\left(x\right)}_{increased\,market\,(+)} \Bigg],$$

where $\alpha_i^p\left(\theta\right)$,$i \in \{h, s, 0\}$ is short notation for $\alpha_i^p\left(r_s\left(\theta\right)\right)$.

C.4. **Theorem 15.**

Proof. We are going to show that there always exists a threshold $\hat{n} > 0$ low enough so that $\Pi_p < \Pi_t\left(r_s^p, \hat{n}\right) \leq \Pi_t$, so we need to show that

$$\int_0^{\hat{n}/\alpha_s^t\left(r_s^p,\hat{n}\right)} \pi_p\left(x\right) \mathrm{d}G\left(x\right) + \int_{\hat{n}/\alpha_s^t\left(r_s^p,\hat{n}\right)}^{+\infty} \pi_p\left(x\right) \mathrm{d}G\left(x\right) < \int_0^{\hat{n}/\alpha_s^t\left(r_s^p,\hat{n}\right)} \pi_c\left(x\right) \mathrm{d}G\left(x\right) + \int_{\hat{n}/\alpha_s^t\left(r_s^p,\hat{n}\right)}^{+\infty} \pi_{t-on}\left(x; r_s^p, \hat{n}\right) \mathrm{d}G\left(x\right).$$

where $\pi_c\left(x\right) = r_h \min\left(k, \alpha_h^c x\right) - c_F$, $\pi_p\left(x\right) = r_h \min\left(k, \alpha_h^p\left(r_s^p\right)x\right) + r_s^p \min\left(k, \alpha_s^p\left(r_s^p\right)x\right) - 2c_F$, and $\pi_{t-on}\left(x; r_s, n\right) = r_h \min\left(k, \alpha_h^t\left(r_s, n\right)x\right) + r_s \min\left(k, \alpha_s^t\left(r_s, n\right)x\right) - 2c_F$. If $c_F > 0$ it is easy to show that $\exists x^\circ > 0 : \pi_c\left(x\right) > \pi_p\left(x\right)\ iff\ x < x^\circ$. If instead $c_F = 0$ but $\alpha_h^c - \alpha_h^p\left(\theta_p\right) > \left(1-\theta_p\right)\alpha_s^p\left(\theta_p\right)$, then this implies that $\pi_p(x) < \pi_c(x)\ \forall x \in (0,k)$ because

$$\pi_p(\theta_p;x) = r_h\alpha_h^p(\theta_p)x + r_h(1-\theta_p)\alpha_s^p(\theta_p)x < r_h\alpha_h^c x = \pi_c(x),$$

hence once again $\exists x^\circ > 0 : \pi_c(x) > \pi_p(x)\ iff\ x < x^\circ$. So we know that $\exists\bar{n} > 0$ such that $\int_0^{n/\alpha_s^t(r_s^p,n)} \pi_p(x)\,\mathrm{d}G(x) < \int_0^{n/\alpha_s^t(r_s^p,n)} \pi_c(x)\,\mathrm{d}G(x)$ for every $n \le \bar{n}$. We are left to show that there exists an $\hat{n} \in (0,\bar{n})$ such that

$$\int_{\hat{n}/\alpha_s^t(r_s^p,\hat{n})}^{+\infty} \left[\pi_{t-on}(x;r_s^p,\hat{n}) - \pi_p(x)\right]\mathrm{d}G(x) > 0.$$

This is ensured if $\lim_{n\to 0^+}\int_z^{+\infty}\left[\pi_{t-on}(x;r_s^p,n) - \pi_p(x)\right]\mathrm{d}G(x) > 0\,\forall z \ge 0$, which holds if and only if $\int_z^{+\infty}\frac{d}{dn}\alpha_h^t(r_s^p,n)\left(1_{x<k\left(\alpha_h^p(r_s^p)\right)^{-1}}r_h - 1_{x<k\left(\alpha_s^p(r_s^p)\right)^{-1}}r_s^p\right)x\mathrm{d}G(x) > 0\,\forall z \ge 0$, which is implied by $\int_z^{+\infty}\left(1_{x<k\left(\alpha_h^p(r_s^p)\right)^{-1}}r_h - 1_{x<k\left(\alpha_s^p(r_s^p)\right)^{-1}}r_s^p\right)x\mathrm{d}G(x) < 0\,\forall z \ge 0$, which is implied by (16.4). □

C.5. **Theorem 16.**

Proof. let $r_s^p = \max_r \Pi_p(r_s^p)$ s.t. $r_s \le r_h$. Then, since $x \le k$ and therefore $\alpha_i^p(r_s) = \alpha_i^t(r_s,n)\ \forall r_s, n > 0$, this means that $r_s^p = \max_{r_s}\left(\alpha_h^p(r_s)r_h + \alpha_s^p(r_s)r\right)$ subject to $r_s \le r_h$. We have two cases:

1. $r_s^p = r_h$. Then $\breve{\pi}_p(r_s^p;\underline{x}) - \breve{\pi}_c(\underline{x}) = \underline{x}\left(\alpha_h^p + \alpha_s^p - \alpha_h^c\right) > 0$ and,

$$\Pi_p - \Pi_t = \int_{\underline{x}}^{n_t/\alpha_s^t(r_s^t,n_t)}\left(\breve{\pi}_p(r_s^p;x) - \breve{\pi}_c(x) - c_F\right)\mathrm{d}G(x) + \int_{n_t/\alpha_s^t(r_s^t,n_t)}^{\bar{x}}\left(\breve{\pi}_p(r_s^p;x) - \breve{\pi}_{t-on}\left(r_s^t,n_t;x\right)\right)\mathrm{d}G(x) =$$

$$= \int_{\underline{x}}^{n_t/\alpha_s^t(r_s^t,n_t)}\left(\breve{\pi}_p(r_h;x) - \breve{\pi}_c(x) - c_F\right)\mathrm{d}G(x) + \int_{n_t/\alpha_s^t(r_s^t,n_t)}^{\bar{x}}\left(\breve{\pi}_p(r_h;x) - \breve{\pi}_{t-on}\left(r_s^t,n_t;x\right)\right)\mathrm{d}G(x) =$$

which is strictly positive if $\breve{\pi}_p(r_h;\underline{x}) - \breve{\pi}_c(\underline{x}) - c_F \ge 0$ or $c_F \le \breve{\pi}_p(r_h;\underline{x}) - \breve{\pi}_c(\underline{x})$, which is a positive number, and where $\left(r_s^t,n_t\right) = \arg\max_{r_s,n}\Pi_t(r_s,n)$ s.t. $r_s < r_h$, $n > 0$.

2. $r_s^p < r_h$. Then td cannot do better than mimic the price of pd and

$$\Pi_p - \Pi_t = \int_{\underline{x}}^{n/\alpha_s^t(r_s^t,n)}\left(\breve{\pi}_p(r_s^p;x) - \breve{\pi}_c(x) - c_F\right)\mathrm{d}G(x) + \int_{n/\alpha_s^t(r_s,n)}^{\bar{x}}\left(\breve{\pi}_p(r_s^p;x) - \pi_t^0\left(r_s^t;x\right)\right)\mathrm{d}G(x) =$$

$$= \int_{\underline{x}}^{n/\alpha_s^t(r_s^p,n)} \left(\breve{\pi}_p\left(r_s^p;x\right) - \breve{\pi}_c\left(x\right) - c_F\right) \mathrm{d}G\left(x\right) + \int_{n/\alpha_s^t(r_s^p,n)}^{\bar{x}} \left(\breve{\pi}_p\left(r_s^p;x\right) - \pi_t^0\left(r_s^p;x\right)\right) \mathrm{d}G\left(x\right) =$$

$$= \int_{\underline{x}}^{n/\alpha_s^t(r_s^p,n)} \left(\breve{\pi}_p\left(r_s^p;x\right) - \breve{\pi}_c\left(x\right) - c_F\right) \mathrm{d}G\left(x\right)$$

which is strictly positive if $\breve{\pi}_p\left(r_s^p;\underline{x}\right) - \breve{\pi}_c\left(\underline{x}\right) - c_F > 0$, or $c_F < \breve{\pi}_p\left(r_s^p;\underline{x}\right) - \breve{\pi}_c\left(x\right)$.

As per the second point in the theorem, consider that for $c_F > \breve{\pi}_p\left(\bar{x}\right) - \breve{\pi}_c\left(\bar{x}\right)$ we have that

$$\Pi_c - \Pi_t = \int_{n/\alpha_s^t(r_s,n)}^{\bar{x}} \left[\breve{\pi}_c\left(x\right) - \breve{\pi}_{t-on}\left(r_s^t, n_t; x\right) + c_F\right] \mathrm{d}G\left(x\right) \geq \int_{n/\alpha_s^t(r_s,n)}^{\bar{x}} \left[\breve{\pi}_c\left(x\right) - \breve{\pi}_p\left(r_s^t; x\right) + c_F\right] \mathrm{d}G\left(x\right) > 0$$

□

C.6. **Additional results for Section 16.**

Lemma 10. *The strategic scarcity effect*

Suppose that $\alpha_h^p\left(r_s\right) > \alpha_s^p\left(r_s\right)$ for some price r_s. Then for every $n > 0$ we have that $\alpha_h^p\left(r_s\right) > \alpha_h^t\left(r_s, n\right) > \alpha_s^t\left(r_s, n\right) > \alpha_s^p\left(r_s\right)$.

Proof. Customer visit equation (C.1) can be rewritten focusing on the value of the visit threshold for those customers with the highest hot period valuation, $\hat{v}_t\left(\bar{v}\right)$, since this uniquely define $\hat{v}_t\left(v_h\right)$, as explained in (Section C.1):

$$\left(\hat{v}_t\left(\bar{v}\right) - r_s\right) = \left(v_h - r_h\right) \frac{\int_{n/\alpha_s^t(\hat{v}_t(\bar{v}))}^{+\infty} \min\left(1, k\left(\alpha_h^t\left(\hat{v}_t\left(\bar{v}\right)\right) x\right)^{-1}\right) \mathrm{d}G_c\left(x\right)}{\int_{n/\alpha_s^t(\hat{v}_t(\bar{v}))}^{+\infty} \min\left(1, k\left(\alpha_s^t\left(\hat{v}_t\left(\bar{v}\right)\right) x\right)^{-1}\right) \mathrm{d}G_c\left(x\right)}.$$

The result is proven by noting that the rhs of the above equation increases in n, and decreases in $\hat{v}_t\left(\bar{v}\right)$, while the lhs increases in $\hat{v}_t\left(\bar{v}\right)$. Hence the same logic applies as in (subsubsection B.7.1). □

Lemma 11. *There exists a discount threshold θ_m such that the operational effect of discounting is positive for all discounts lower than θ_m and negative otherwise. Formally, $\Pi'_{p-op}\left(\theta\right) > 0\, iff\, \theta \leq \theta_m$. It follows that, if the operational effect of discounting is positive for a certain discount level θ, then the total operational effect of discounting on profit is also positive, i.e.*

$$\Pi'_{p-op}\left(\theta\right) > 0 \Rightarrow \int_0^{\theta} \Pi'_{p-op}\left(\tau\right) \mathrm{d}\tau > 0.$$

Proof. We show that $\exists\ddot{\theta}:\int_0^{+\infty}\frac{d\alpha_h^p(\theta)}{d\theta}r_h\left(1_{x<k\left(\alpha_h^p(\theta)\right)^{-1}}-1_{x<k\left(\alpha_s^p(\theta)\right)^{-1}}(1-\theta)\right)x\mathrm{d}G(x)>0\,iff\,\theta\leq\ddot{\theta}$, which is stronger than the above condition. We prove this considering two cases.

First, suppose that $\int_0^{+\infty}\frac{d\alpha_h^p(\theta)}{d\theta}r_h\left(1_{x<k\left(\alpha_h^p(\theta)\right)^{-1}}-1_{x<k\left(\alpha_s^p(\theta)\right)^{-1}}(1-\theta)\right)x\mathrm{d}G(x)>0$ for a given θ; then it must be that $\alpha_h^p(\theta)>\alpha_s^p(\theta)$. Suppose not. Then $\left(1_{x<k\left(\alpha_h^p(\theta)\right)^{-1}}-1_{x<k\left(\alpha_s^p(\theta)\right)^{-1}}(1-\theta)\right)>0$ for every x and since $\frac{d\alpha_h^p(\theta)}{d\theta}<0$ then the integral cannot be positive. So $\alpha_h^p(\theta)>\alpha_s^p(\theta)$, and the condition above implies that $\int_0^{k\left(\alpha_h^p(\theta)\right)^{-1}}r_h\theta x\mathrm{d}G(x)<\int_{k\left(\alpha_h^p(\theta)\right)^{-1}}^{k\left(\alpha_s^p(\theta)\right)^{-1}}r_h(1-\theta)x\mathrm{d}G(x)$. If θ decreases, I have that LHS shrinks and RHS grows, so the condition must still hold.

Otherwise, suppose that $\int_0^{+\infty}\frac{d\alpha_h^p(\theta)}{d\theta}r_h\left(1_{x<k\left(\alpha_h^p(\theta)\right)^{-1}}-1_{x<k\left(\alpha_s^p(\theta)\right)^{-1}}(1-\theta)\right)x\mathrm{d}G(x)<0$ for some θ; then we have two subcases.

If $\alpha_h^p(\theta)>\alpha_s^p(\theta)$, then $\int_0^{k\left(\alpha_h^p(\theta)\right)^{-1}}r_h\theta x\mathrm{d}G(x)>\int_{k\left(\alpha_h^p(\theta)\right)^{-1}}^{k\left(\alpha_s^p(\theta)\right)^{-1}}r_h(1-\theta)x\mathrm{d}G(x)$: if θ increases, I have that LHS grows and RHS shrinks as long as $\alpha_h^p(\theta)>\alpha_s^p(\theta)$; when it no longer holds, see the next subcase.

If instead $\alpha_h^p(\theta)<\alpha_s^p(\theta)$, then $\left(1_{x<k\left(\alpha_h^p(\theta)\right)^{-1}}-1_{x<k\left(\alpha_s^p(\theta)\right)^{-1}}(1-\theta)\right)>0$ for every x, hence it becomes negative once it is integrated and multiplied by $\frac{d\alpha_h^p(\theta)}{d\theta}<0$. □

Lemma 12. *Condition $\Pi_{p-op}^{'}(\theta_p)>0$ implies that the hot period is busier than slow, and subject to capacity shortages.*

Proof. Suppose that more customers visit in the slow period than in the hot period.

Then $\int_0^{+\infty}\left(1_{x<k\left(\alpha_h^p(\theta)\right)^{-1}}-1_{x<k\left(\alpha_s^p(\theta)\right)^{-1}}(1-\theta)\right)x\mathrm{d}G(x)>0\,\forall\theta$, hence $\Pi_{p-op}^{'}(\theta_p)<0$. Contradiction. Now, suppose that more customers visit in the hot period than in the slow period, but there are no capacity shortages. Then, again, $\Pi_{p-op}^{'}(\theta_p)>0$. Contradiction. □

Lemma 13. *Concavity of the three effects of discount on profit under price discrimination*

- *The increased market effect is decreasing in θ iff $\frac{d}{d\theta^2}\alpha_0^p\geq-b_{im}$, where b_{im} is a positive number;*
- *The operational effect is decreasing in θ iff $|\frac{d}{d\theta^2}\alpha_h^p|<b_{op}$, where b_{op} is a positive number;*
- *The margin effect is always decreasing in θ.*

Proof. Note that $\frac{d}{d\theta}\Pi'_{p-im}(\theta)<0$ iff

$$\underbrace{-\frac{d}{d\theta}\left(\frac{k}{\alpha_s^p}\right)\left[r_h\left(1-\theta\right)\frac{d}{d\theta}\alpha_0^p\frac{k}{\alpha_p^p}g\left(\frac{k}{\alpha_p^p}\right)\right]}_{<0}+\int\limits_0^{k/\alpha_s^p}\left(\underbrace{r_h\frac{d}{d\theta}\alpha_0^p}_{<0}-r_h\left(1-\theta\right)\frac{d^2}{d\theta^2}\alpha_0^p\right)x\mathrm{d}G\left(x\right)<0,$$

and not also that $\frac{d}{d\theta}\Pi'_{p-op}(\theta)<0$ iff

$$\frac{d^2}{d\theta^2}\alpha_h^p r_h\left[\int\limits_0^{k/\alpha_h^p}\theta x\mathrm{d}G\left(x\right)+\int\limits_{k/\alpha_h^p}^{k/\alpha_s^p}\left(1-\theta\right)x\mathrm{d}G\left(x\right)\right]+\frac{d}{d\theta}\alpha_h^p r_h\left[\int\limits_0^{k/\alpha_s^p}x\mathrm{d}G\left(x\right)+\frac{k}{\alpha_s^p}g\left(\frac{k}{\alpha_s^p}\right)\left(\frac{d}{d\theta}\frac{k}{\alpha_h^p}-\frac{d}{d\theta}\frac{k}{\alpha_s^p}\right)\right],$$

where the second component is always negative. That $\frac{d}{d\theta}\Pi'_{p-mg}(\theta)<0$ is then easy to check. $\square$

C.7. **Profit under threshold discounting for a mixed population of customers–extended model.**

$$\Pi_t^\gamma=\max_{r_s,n}\left[r_h\int\limits_0^{n/\alpha_s^t(r_s,n|\gamma)}\left(\min(k,\alpha_h^c x)-c_F\right)\,\mathrm{d}G(x)+\right.$$

(C.3)

$$\left.+\int\limits_{n/\alpha_s^t(r_s,n|\gamma)}^{+\infty}\left(\min\left(k,\alpha_h^t\left(r_s,n;\gamma\right)x\right)\left(r_h-c\right)+\min\left(k,\alpha_s^t\left(r_s,n;\gamma\right)x\right)\left(r_s-c\right)-2c_F\right)\,\mathrm{d}G(x)\right],$$

$$\textit{subject to}\, r_s<r_h,\; n>0,$$

where $\alpha_s^t\left(r_s,n;\gamma\right)=\gamma\alpha_{s,\gamma}^t\left(r_s,n\right)+\left(1-\gamma\right)\alpha_s^t\left(r_s;\mu\right)$ is the fraction of the population that in equilibrium subscribes and visits the firm during the slow period when the deal is active, with $\alpha_s^t\left(r_s;\mu\right)=\int_0^{\bar v}\int_{r_s+(\tau_h-r_h)^+}^{\bar v}h\left(\tau_h,\tau_s\right)\mathrm{d}\tau_s\mathrm{d}\tau_h$ and $\alpha_{s,\gamma}^t\left(r_s,n\right)=\int_0^{\bar v}\int_{r_s+\left(\left(\hat v_t(\bar v;r_s,n)-r_s\right)\left(\tau_h-r_h\right)\left(\bar v-r_h\right)^{-1}\right)^+}^{\bar v}h\left(\tau_h,\tau_s\right)\mathrm{d}\tau_s\mathrm{d}\tau_h$, with $\hat v_t\left(\bar v;r_s,n\right)$ being the solution to

(C.4)

$$\frac{\left(\hat v_t(\bar v)-r_s\right)}{\left(v_h-r_h\right)}=\frac{\int_{n/\left(\gamma\alpha_{s,\gamma}^t\left(\hat v_t(\bar v)\right)+\bar\gamma\alpha_s^t\left(r_s;\mu\right)\right)}^{+\infty}\min\left(1,k\left(\left(1-\gamma\alpha_{s,\gamma}^t\left(\hat v_t(\bar v)\right)-\bar\gamma\alpha_s^t\left(r_s;\mu\right)-\alpha_0^t\left(r_s\right)\right)x\right)^{-1}\right)\mathrm{d}G_c(x)}{\int_{n/\left(\gamma\alpha_{s,\gamma}^t\left(\hat v_t(\bar v)\right)+\bar\gamma\alpha_s^t\left(r_s;\mu\right)\right)}^{+\infty}\min\left(1,k\left(\left(\gamma\alpha_{s,\gamma}^t\left(\hat v_t(\bar v)\right)+\bar\gamma\alpha_s^t\left(r_s;\mu\right)\right)x\right)^{-1}\right)\mathrm{d}G_c(x)},$$

where $\bar\gamma=1-\gamma$, $\alpha_0^t\left(r_s\right)=\int_0^{r_h}\int_0^{r_s}h\left(\tau_h,\tau_s\right)\mathrm{d}\tau_s\mathrm{d}\tau_h$, and with the understanding that $\hat v_t\left(\bar v\right)$ is short notation for $\hat v_t\left(\bar v;r_s,n\right)$, and that it uniquely identifies the slow period valuation $\hat v_t\left(v_h;r_s,n\right)$ that makes a strategic customer with hot period valuation v_h indifferent between strategies (s,vs,vh) and

(ns, vh, vh), this being the straight line that connects (r_h, r_s) with $\hat{v}_t(\bar{v}; r_s, n)$. Clearly, $\alpha_h^t(r_s, n; \gamma) = 1 - \alpha_s^t(r_s, n; \gamma) - \alpha_0^t(r_s)$.

C.8. **Theorem 17.**

Proof. Note that

$$\Pi_p = \int_0^{+\infty} \left[r_h \min\left(k, \alpha_h^p(r_s^p, \gamma)\, x\right) + r_s^p \min\left(k, \alpha_s^p(r_s^p, \gamma)\, x\right)\right] \mathrm{d}G(x)$$

$$\frac{\partial}{\partial\gamma}\Pi_p = \int_0^{+\infty} \left[r_h \frac{\partial}{\partial\gamma}\min\left(k, \alpha_h^p(r_s^p, \gamma)\, x\right) + r_s^p \frac{\partial}{\partial\gamma}\min\left(k, \alpha_s^p(r_s^p, \gamma)\, x\right)\right] \mathrm{d}G(x) =$$

$$= \frac{\partial}{\partial\gamma}\alpha_h^p(r_s^p, \gamma) \int_0^{+\infty} \left(1_{\{\alpha_h(r_s^p,\gamma)x<k\}} r_h - 1_{\{\alpha_s(r_s^p,\gamma)x<k\}} r_s^p\right) x \mathrm{d}G(x)$$

because $\frac{\partial}{\partial\gamma}\alpha_h^p = -\frac{\partial}{\partial\gamma}\alpha_s^p$, since $\frac{\partial}{\partial\gamma}\alpha_0^p = 0$. From (16.4) we have $\int_0^m \left(1_{x<k\left(\alpha_h^p(r_s^p)\right)^{-1}}\{r_h\} - r\right) x \mathrm{d}G(x) < 0$, hence $\frac{\partial}{\partial\gamma}\alpha_h^p(r_s^p, \gamma) \int_0^{+\infty} \left[1_{\{\alpha_h(r_s^p,\gamma)x<k\}} r_h - 1_{\{\alpha_s(r_s^p,\gamma)x<k\}} r_s^p\right] x \mathrm{d}G(x) > 0$ because $\frac{\partial}{\partial\gamma}\alpha_h^p < 0$. From the envelope theorem $\frac{d}{d\gamma}\Pi_p(r_s^p, \gamma) = \frac{\partial}{\partial\gamma}\Pi_p$. Finally, note that from the foc of PD and TD wrt r_s we know that $(\gamma = 0)$:

$$\int_0^{+\infty} \left[\alpha_s(r_s^p) - \frac{d\alpha_0(r_s^p)}{dr_s} r_s^p\right] x \mathrm{d}G(x) + \int_0^{+\infty} \left[\frac{d\alpha_h(r_s^p)}{dr_s}\left(1_{x<k\alpha_h^{-1}(r_s^p)}\{r_h\} - r_s^p\right)\right] x \mathrm{d}G(x) = 0$$

$$\int_{n^t/\alpha_s(r_s^t)}^{+\infty} \left[\alpha_s(r_s^t) - \frac{d\alpha_0(r_s^t)}{dr_s} r_s^t\right] x \mathrm{d}G(x) + \int_{n^t/\alpha_s(r_s^t)}^{+\infty} \left[\frac{d\alpha_h(r_s^t)}{dr_s}\left(1_{x<k\alpha_h^{-1}(r_s^t)}\{r_h\} - r_s^t\right)\right] x \mathrm{d}G(x) = 0$$

Condition (16.4) implies that $\int_0^{+\infty} \left[\frac{d\alpha_s(r_s^p)}{dr_s}\left(1_{x<k\alpha_h^{-1}(r_s^p)}\{r_h\} - r_s^p\right)\right] x \mathrm{d}G(x) < 0$, which implies that $\int_0^{+\infty} \left[\alpha_s(r_s^p) + \frac{d\alpha_0(r_s^p)}{dr_s} r_s^p\right] x \mathrm{d}G(x) > 0$, so that $\left[\alpha_s(r_s^p) + \frac{d\alpha_0(r_s^p)}{dr_s} r_s^p\right] \mathbb{E}[x] > 0$, which implies that $\alpha_s(r_s^p) + \frac{d\alpha_0(r_s^p)}{dr_s} r_s^p > 0$. If $r_s^t < r_s^p$ then from regularity assumptions it follows that $\alpha_s(r_s^t) + \frac{d\alpha_0(r_s^t)}{dr_s} r_s^t > 0$, hence $\left[\alpha_s(r_s^t) + \frac{d\alpha_0(r_s^t)}{dr_s} r_s^t\right] \int_{n^t/\alpha_s(r_s^t)}^{+\infty} x \mathrm{d}G(x) > 0$. Clearly, this therefore implies that $\int_{n^t/\alpha_s(r_s^t)}^{+\infty} \left[\frac{d\alpha_h(r_s^t)}{dr_s}\left(1_{x<k\alpha_h^{-1}(r_s^t)}\{r_h\} - r_s^t\right)\right] x \mathrm{d}G(x) < 0$. If instead $r_s^t > r_s^p$ note that

$$\int_0^{+\infty} \left[\frac{d\alpha_h(r_s^p)}{dr_s}\left(1_{x<k\alpha_h^{-1}(r_s^p)}\{r_h\} - 1_{x<k\alpha_s^{-1}(r_s^p)} r_s^p\right)\right] x \mathrm{d}G(x) =$$

$$= \frac{d\alpha_h\left(r_s^p\right)}{dr_s}\left(\int_0^{k/\alpha_h\left(r_s^p\right)}\left(r_h - r_s^p\right)x\mathrm{d}G\left(x\right) + \int_{k/\alpha_h\left(r_s^p\right)}^{k/\alpha_s\left(r_s^p\right)}\left(-r_s^p\right)x\mathrm{d}G\left(x\right)\right) < 0$$

and $\int_0^{k/\alpha_h(r_s)}\left(r_h - r\right)x\mathrm{d}G\left(x\right) + \int_{k/\alpha_h(r_s)}^{k/\alpha_s(r_s)}\left(-r\right)x\mathrm{d}G\left(x\right)$ is decreasing in r. Hence, it must be that $\int_{n^t/\alpha_s(r_s^t)}^{+\infty}\left[\frac{d\alpha_h\left(r_s^t\right)}{dr_s}\left(1_{x<k\alpha_h^{-1}(r_s^t)}\left\{r_h\right\} - r_s^t\right)\right]x\mathrm{d}G\left(x\right) < 0$.

Now take $\gamma > 0$, and let $\left(r_s^{t\mu}, n_t^\mu\right)$ be the profit maximizing deal for the non-strategic case. Then $\exists\Delta n(\gamma)$ such that $\hat{x}_t\left(r_s^{t\mu}, n_t^\mu, 0\right) \triangleq \left(n_t^\mu\right)\alpha_s^t\left(r_s^{t\mu}, n_t^\mu, 0\right)^{-1} = \left(n_t^\mu + \Delta n\left(\gamma\right)\right)\alpha_s^t\left(r_s^{t\mu}, n_t^\mu + \Delta n\left(\gamma\right), \gamma\right)^{-1} = \hat{x}_t\left(r_s^{t\mu}, n_t^\mu, \gamma\right)$ and $\alpha_s^t\left(r_s^{t\mu}, n_t^\mu, 0\right) < \alpha_s^t\left(r_s^{t\mu}, n_t^\mu + \Delta n\left(\gamma\right), \gamma\right)$. Hence, for $\gamma > 0$ small enough, we have that $\Pi_t^\mu\left(r_s^{t\mu}, n_t^\mu\right) < \Pi_t^\gamma\left(r_s^{t\mu}, n_t^\mu + \Delta n\left(\gamma\right)\right) \leq \Pi_t^\gamma$. $\square$

C.9. **Transaction Cost of Subscription.**

Equilibrium conditions for the continuation game $\Gamma\left(r_s, n\right)$ *after the announcement* $\left(r_s, n\right)$*.* The equilibrium is similar to the one described in Section C.1, and is graphically depicted in Figure C.1. Specifically, an equilibrium to the customer continuation game $\Gamma\left(r_s, n\right)$ is fully characterized by the vector $\left(\dot{v}_\phi, \ddot{v}_\phi\right)$, whose components represent the highest slow period valuation of customers who do not visit the firm, which is equal to r_s under threshold discouning, and the highest slow period valuation of customers who visit the firm in the hot period when the deal is active, which corresponds to $\hat{v}_t$ under threshold discounting. The vector $\left(\dot{v}_\phi, \ddot{v}_\phi\right)$, the fraction of customers that subscribe, $\alpha_{s,\phi}^t$, and the fraction of customers that do not visi the firm, $\alpha_{0,\phi}^t$, must satisfy the equilibrium conditions of the continuation game $\Gamma\left(r_s, n\right)$ for a given announcement $\left(r_s, n\right)$ of the firm

$$\text{(C.5)}\qquad \begin{cases} \dot{v}_\phi = r_s + \frac{\phi}{\int_{n/\alpha_{s,\phi}}^{+\infty}\min\left(1, k/\alpha_{s,\phi}\right)\mathrm{d}G_c(x)} \\ \ddot{v}_\phi = r_s + \left(\bar{v} - r_h\right)\frac{\int_{n/\alpha_{s,\phi}}^{+\infty}\min\left(1, k/\left(1-\alpha_{s,\phi}-\alpha_{0,\phi}\right)\right)\mathrm{d}G_c(x)}{\int_{n/\alpha_{s,\phi}}^{+\infty}\min\left(1, k/\alpha_{s,\phi}\right)\mathrm{d}G_c(x)} + \frac{\phi}{\int_{n/\alpha_{s,\phi}}^{+\infty}\min\left(1, k/\alpha_{s,\phi}\right)\mathrm{d}G_c(x)} \\ A_s\left(\dot{v}_\phi, \ddot{v}_\phi\right) = \alpha_{s,\phi}^t \\ A_0\left(\dot{v}_\phi\right) = \alpha_{0,\phi}^t \end{cases}$$

where $\ddot{v}$ is short notation for $\ddot{v}\left(\bar{v}\right)$, with the understanding that $\ddot{v}\left(v_h\right)$ directly follows, and where $A_s\left(\dot{v}, \ddot{v}\right) \triangleq \int_0^{\bar{v}}\left(\int_{\dot{v}+((\ddot{v}-\dot{v})(\tau_h-r_h)/(\bar{v}-r_h))^+}^{\bar{v}} h\left(\tau_h, \tau_s\right)\mathrm{d}\tau_s\right)\mathrm{d}\tau_h$ and $A_0\left(\dot{v}\right) \triangleq \int_0^{r_h}\left(\int_0^{\dot{v}} h\left(\tau_h, \tau_s\right)\mathrm{d}\tau_s\right)\mathrm{d}\tau_h$ are functions that compute the fraction of customers that subscribe and visit on slow when the deal

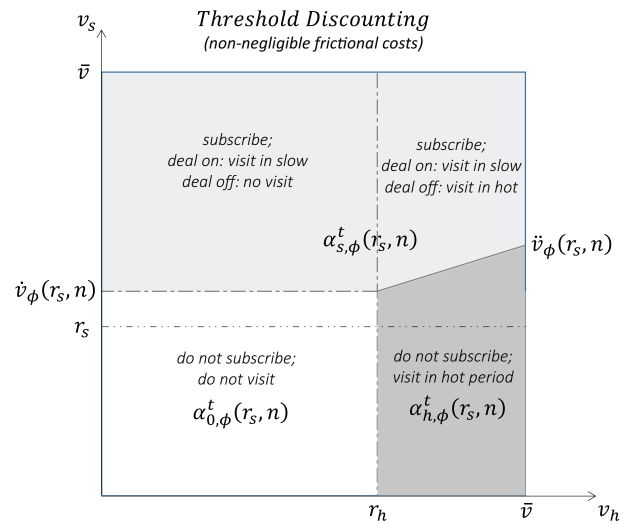

FIGURE C.1: Customer subscription and visit strategy under threshold discounting when cusotomers incur a non-negligible subcription cost

is active, and that do not subscribe and do not visit the firm, respectively, as defined geometrically by $\dot{v}$ and $\ddot{v}$. The remaining fraction of customers is represented by $\alpha^t_{h,\phi}$. The equilibrium strategies for the non-negligible transaction cost case for customers belonging to the three groups $\alpha^t_{j,\phi}$ are the same as per the case of customers belonging to α^t_j in the negligible transaction costs case.

C.10. **Theorem 19.**

Proof. The first point is simply proven because $\dot{v}_\phi > r_s$ from (C.5). To show that the case of non-negligible transaction costs leads to the firm serving fewer customers in the slow period, note that it must be $\alpha^t_s(r_s, n; \phi) < \alpha^t_s(r_s, n)$. Suppose not. Then it must be that $\alpha^t_h(r_s, n; \phi) > \alpha^t_h(r_s, n)$ because $\alpha^t_0(r_s, n; \phi) > \alpha^t_0(r_s, n)$.

Then it must be that $\frac{\int_{n/\alpha^t_s(r_s,n;\phi)}^{+\infty} \min\left(1, k/\alpha^t_h(r_s,n;\phi)\right)\mathrm{d}G(x)}{\int_{n/\alpha^t_s(r_s,n;\phi)}^{+\infty} \min(1, k/\alpha^t_s(r_s,n;\phi))\mathrm{d}G(x)} > \frac{\int_{n/\alpha^t_s(r_s,n)}^{+\infty} \min\left(1, k/\alpha^t_h(r_s,n)\right)\mathrm{d}G(x)}{\int_{n/\alpha^t_s(r_s,n)}^{+\infty} \min(1, k/\alpha^t_s(r_s,n))\mathrm{d}G(x)}$ because the ratio $\frac{\int_{n/\alpha_s}^{+\infty} \min(1, k/\alpha_h)\mathrm{d}G(x)}{\int_{n/\alpha_s}^{+\infty} \min(1, k/\alpha_s)\mathrm{d}G(x)}$ increases in α_s and decreases in α_h. Noting that

$$\ddot{v}_\phi = \dot{v}_\phi + (\bar{v} - r_h) \frac{\int_{n/\alpha^t_s(r_s,n;\phi)}^{+\infty} \min\left(1, k/\alpha^t_h(r_s, n; \phi)\right) \mathrm{d}G(x)}{\int_{n/\alpha^t_s(r_s,n;\phi)}^{+\infty} \min\left(1, k/\alpha^t_s(r_s, n; \phi)\right) \mathrm{d}G(x)} >$$

$$> r_s + (\bar{v} - r_h) \frac{\int_{n/\alpha^t_s(r_s,n)}^{+\infty} \min\left(1, k/\alpha^t_h(r_s, n)\right) \mathrm{d}G(x)}{\int_{n/\alpha^t_s(r_s,n)}^{+\infty} \min\left(1, k/\alpha^t_s(r_s, n)\right) \mathrm{d}G(x)} = \ddot{v}_t$$

and that $\dot{v}_\phi > r$, we reach the conclusion that $A_s(r_s, \ddot{v}_t) > A_s(\dot{v}_\phi, \ddot{v}_\phi)$. Contradiction.

To show that the case of non-negligible transaction costs leads to the firm serving more customers in the hot period, note that the set of equations defining the equilibrium in $\Gamma(r_s, n; \phi)$ for a given

r and n is:

(C.6)

$$\begin{cases} \dot{v}_\phi = r_s + \frac{\phi}{\int_{n/\alpha_s^t(r_s,n;\phi)}^{+\infty} \min(1,k/\alpha_s^t(r_s,n;\phi))\mathrm{d}G_c(x)} \\ \ddot{v}_\phi = r_s + \frac{\phi}{\int_{n/\alpha_s^t(r_s,n;\phi)}^{+\infty} \min(1,k/\alpha_s^t(r_s,n;\phi))\mathrm{d}G_c(x)} + (\bar{v} - r_h) \frac{\int_{n/\alpha_s^t(r_s,n;\phi)}^{+\infty} \min\left(1,k/\alpha_h^t(r_s,n;\phi)\right)\mathrm{d}G_c(x)}{\int_{n/\alpha_s^t(r_s,n;\phi)}^{+\infty} \min(1,k/\alpha_s^t(r_s,n;\phi))\mathrm{d}G_c(x)} \\ A_s\left(\dot{v}_\phi, \ddot{v}_\phi\right) = \alpha_s^t\left(r_s, n; \phi\right) \\ A_h\left(\dot{v}_\phi, \ddot{v}_\phi\right) = \alpha_h^t\left(r_s, n; \phi\right) \end{cases}.$$

Then define $\Delta r \triangleq \min\left(\bar{v} - r_s, \frac{\phi}{\int_{n/\alpha_s^t(r_s,n;\phi)}^{+\infty} \min(1,k/\alpha_s^t(r_s,n;\phi))\mathrm{d}G(x)}\right)$ and consider a price increase of Δr in the case of negligible transaction costs; the equilibrium in $\Gamma\left(r_s + \Delta r, n\right)$ would be given by:

$$\begin{cases} \dot{v}_t = r_s + \Delta r \\ \ddot{v}_t = r_s + \Delta r + (\bar{v} - r_h) \frac{\int_{n/\alpha_s^t(r_s+\Delta r,n)}^{+\infty} \min\left(1,k/\alpha_h^t(r_s+\Delta r,n)\right)\mathrm{d}G_c(x)}{\int_{n/\alpha_s^t(r_s+\Delta r,n)}^{+\infty} \min(1,k/\alpha_s^t(r_s+\Delta r,n))\mathrm{d}G_c(x)} \\ A_s\left(\dot{v}_t, \ddot{v}_t\right) = \alpha_s^t\left(r_s + \Delta r, n\right) \\ A_h\left(\dot{v}_t, \ddot{v}_t\right) = \alpha_h^t\left(r_s + \Delta r, n\right) \end{cases},$$

which mirrors exactly the set of equations for $\Gamma\left(r_s, n; \phi\right)$, leading to $\alpha_s^t\left(r_s + \Delta r, n\right) = \alpha_s^t\left(r_s, n; \phi\right)$ and $\alpha_h^t\left(r_s + \Delta r, n\right) = \alpha_h^t\left(r_s, n; \phi\right)$. Periods are substitute goods hence $\alpha_h^t\left(r_s + \Delta r, n\right) > \alpha_h^t\left(r_s, n\right)$, therefore $\alpha_h^t\left(r_s, n; \phi\right) > \alpha_h^t\left(r_s, n\right)$.

The third point is proven by showing that there exists $\left(\hat{r}_s, \hat{n}\right)$ such that $\Pi_t\left(\hat{r}_s, \hat{n}\right) > \Pi_t\left(r_{s\phi}^t, n_{t\phi}; \phi\right)$, with $\left(r_{s\phi}^t, n_{t\phi}\right) = \arg\max_{r_s,n} \Pi_t\left(r_s, n; \phi\right)$. Specifically, let $\hat{r}_s = \dot{v}_\phi\left(r_{s\phi}^t, n_{t\phi}\right)$ and let $\hat{n} = n_{t\phi}$. Then, with these two conditions in mind, it is easy to see that the equilibrium conditions in customer continuation game Γ in the two cases are identical, i.e.

$$\begin{cases} \hat{v}_t = \hat{r}_s + (\bar{v} - r_h) \frac{\int_{\hat{n}/\alpha_s^t}^{+\infty} \min\left(1,k/\alpha_s^t\right)\mathrm{d}G_c(x)}{\int_{\hat{n}/\alpha_s^t}^{+\infty} \min(1,k/\alpha_s^t)\mathrm{d}G_c(x)} \\ A_s\left(\hat{r}_s, \hat{v}_t\right) = \alpha_s \\ A_0\left(\hat{r}_s\right) = \alpha_0 \end{cases} \quad \begin{cases} \ddot{v}_\phi = \dot{v}_\phi\left(r_{s\phi}^t, n_{t\phi}\right) + (\bar{v} - r_h) \frac{\int_{n/\alpha_{s,\phi}}^{+\infty} \min\left(1,k/\left(1-\alpha_{s,\phi}-\alpha_{0,\phi}\right)\right)\mathrm{d}G_c(x)}{\int_{n/\alpha_{s,\phi}}^{+\infty} \min\left(1,k/\alpha_{s,\phi}\right)\mathrm{d}G_c(x)} \\ A_s\left(\dot{v}_\phi, \ddot{v}_\phi\right) = \alpha_{s,\phi} \\ A_0\left(\dot{v}_\phi\right) = \alpha_{0,\phi} \end{cases},$$

□

hence will lead to the same sales in every period and for every market size realization, but the firm will earn higher margins in the negligible transaction cost case, appropriating the transaction cost that customers do not incur in the form of higher margins.

When $\phi \to 0^+$, C.6 converges to the conditions for the equilibrium in the negligible transaction cost case.

C.11. **Mediated Threshold Discounting.** Define

$$\Pi_2(r_s,n|\eta) \triangleq \int_0^{n/\alpha_s^t(r_s,n)} r_h \min(k,\alpha_h x)\,\mathrm{d}G(x) + \int_{n/\alpha_s^t(r_s,n)}^{+\infty} \left[r_h \min\left(k,\alpha_h^t(r_s,n)\,x\right) + (1-\eta)\,r_s \min\left(k,\alpha_s^t(r_s,n)\,x\right)\right]\mathrm{d}G(x)$$

On the threshold:

$$\frac{d}{dn}\Pi_2(r_s,n|\eta) = \frac{d}{dn}\Pi_{IN}(r_s,n|1-\eta) + \underbrace{\int_{n/\alpha_s^t(r_s,n)}^{+\infty} \left[r_h \mathbf{1}_{\alpha_h^t(r_s,n)x<k} \frac{d}{dn}\alpha_h^t(r_s,n)\,x\right]\mathrm{d}G(x)}_{<0} +$$

$$\underbrace{+\frac{d}{dn}\left(\frac{n}{\alpha_s^t(r_s,n)}\right) r_h g\left(\frac{n}{\alpha_s^t(r_s,n)}\right)\left[\min\left(k,\alpha_h \frac{n}{\alpha_s^t(r_s,n)}\right) - \min\left(k,\alpha_h^t \frac{n}{\alpha_s^t(r_s,n)}\right)\right]}_{>0}$$

$$\frac{d}{dn}\Pi_{t-med}(r_s,n|\eta) = \frac{d}{dn}\Pi_2(r_s,n|\eta) + \underbrace{\frac{d}{dn}\left(\frac{n}{\alpha_s^t(r_s,n)}\right)F}_{>0} > \frac{d}{dn}\Pi_2(r_s,n|\eta).$$

If $\gamma = 0$ then $\frac{d}{dn}\alpha_s^t(r_s,n) = 0$, hene $\frac{d}{dn}\Pi_{t-med}(r_s,n|\eta) > \frac{d}{dn}\Pi_{IN}(r_s,n|1-\eta)$, whih implies

$$n_t(r_s) > \underset{\substack{n \\ s.t.\, n>0}}{\arg\max}\, \Pi_{IN}(r_s,n|1-\eta) = n_s^{IN}(r_s),$$

sine $n_s^{IN}(r_s)$ is the same for every positive η. It also follows that $\forall r_s \exists \bar{\gamma}(r_s) > 0 : \forall \gamma \leq \bar{\gamma}(r_s)$ we have $n_t(r_s) > n_s^{IN}(r_s)$.

As per the pricing decision, note that

$$\frac{d}{dr}\Pi_2(r_s,n|\eta) = \frac{d}{dr}\Pi_{IN}(r_s,n|1-\eta) +$$

$$+\frac{d}{dr}\int_{n/\alpha_s^t(r_s,n)}^{+\infty} \left[r_h \min\left(k,\alpha_h^t(r_s,n)\,x\right)\right]\mathrm{d}G(x) + \frac{d}{dr}\int_0^{n/\alpha_s^t(r_s,n)} \left[r_h \min(k,\alpha_h x)\right]\mathrm{d}G(x) =$$

$$= \frac{d}{dr}\Pi_{IN}(r_s,n|1-\eta) +$$

$$+\underbrace{\frac{d}{dr}\left(\frac{n}{\alpha_s^t(r_s,n)}\right) r_h g\left(\frac{n}{\alpha_s^t(r_s,n)}\right)\left[\min\left(k,\alpha_h\frac{n}{\alpha_s^t(r_s,n)}\right)-\min\left(k,\alpha_h^t(r_s,n)\frac{n}{\alpha_s^t(r_s,n)}\right)\right]}_{>0}+$$

$$+\underbrace{\int\limits_{n/\alpha_s^t(r_s,n)}^{+\infty}\left[r_h 1_{x<k/\alpha_s^t(r_s,n)}\frac{d}{dr}\alpha_h^t(r_s,n)\,x\right]\mathrm{d}G(x)}_{>0}>\frac{d}{dr}\Pi_{IN}(r_s,n|1-\eta),$$

and

$$\frac{d}{dr}\Pi_{t-med}(r_s,n|\eta)=\frac{d}{dr}\Pi_2(r_s,n|\eta)+\underbrace{\frac{d}{dr}\left(\frac{n}{\alpha_s^t(r_s,n)}\right)_F}_{>0}>\frac{d}{dr}\Pi_2(r_s,n|\eta)$$

hene, for every $n>0$,

$$r_s^t(n)>\arg\max_{r_s}\Pi_{IN}(r_s,n|1-\eta)=r_s^{IN}(n)$$

sine $r_s^{IN}(n)$ is the same for every positive η, and the result is proven.

Finally, that the service provider earns a lower profit is proven by taking the optimal deal for the intermediary, $\left(r_s^{IN},n^{IN}\right)$, and noting that, thanks to our first result, the service provider would earn more with the deal $\left(r_s^{IN}+\epsilon,n^{IN}\right)$ where ϵ is a positive and small enough number.

C.12. **Additional results for Section 17.**

Lemma 14. *Equivalence between higher fraction of strategic customers and higher demand smoothing under threshold discounting*

Consider a firm employing threshold discounting, and suppose that the hot period is busier than the slow period. For any given $\gamma_1,\gamma_2>0$, $\gamma_1<\gamma_2$, and for any given r_s and $n_2>0$, there exists a $n_1\in(0,n_2)$ such that the deal (r_s,n_2) leads to the same market size trigger level, $n_1/\alpha_s^t(r_s,n_1)=n_2/\alpha_s^t(r_s,n_2)$, and the same fraction of total visitors, $\alpha_h^t(r_s,n_1)+\alpha_s^t(r_s,n_1)=\alpha_h^t(r_s,n_2)+\alpha_s^t(r_s,n_2)$, but to fewer visitors in the slow period, $\alpha_s^t(r_s,n_1)>\alpha_s^t(r_s,n_2)$.

Proof. Let's begin by noting that, by construction, $n_1=n_2*\alpha_s^t(r_s,n_1,\gamma_1)\,\alpha_s^t(r_s,n_2,\gamma_2)^{-1}$ implies that $n_1\alpha_s^t(r_s,n_1,\gamma_1)^{-1}=n_2\alpha_s^t(r_s,n_2,\gamma_2)^{-1}$. The existence of a solution is due to the fact that $\frac{n}{\alpha_s^t(r_s,n,\gamma)}$ is increasing in n, for the same reasons as for the base model. It remains to show that $\alpha_s^t(r_s,n_1,\gamma_1)>\alpha_s^t(r_s,n_2,\gamma_2)$. Consider that α_h^t and α_s^t are uniquely defined–geometrically–by $\hat{v}_t(v_h;r_s,n)$ and r_s, hence customer subscription equations can be written as

$$\frac{\hat{v}_t\left(v_h;r_s,n_1\right)-r_s}{v_h-r_h}=\frac{\int_{n_1/\alpha_s^t(\hat{v}_t(v_h;r_s,n_1);r_s)}^{+\infty}\min\left(1,k\left(\left(\gamma_1\alpha_h^t(\hat{v}_t(v_h;r_s,n_1)\,;r_s)+\bar{\gamma}_1\alpha_h^\mu(r_s)\right)x\right)^{-1}\right)\mathrm{d}G\left(x\right)}{\int_{n_1/\alpha_s^t(\hat{v}_t(v_h;r_s,n_1);r_s)}^{+\infty}\min\left(1,k\left(\left(\gamma_1\alpha_s^t(\hat{v}_t(v_h;r_s,n_1)\,;r_s)+\bar{\gamma}_1\alpha_s^\mu(r_s)\right)x\right)^{-1}\right)\mathrm{d}G\left(x\right)}$$

$$\frac{\hat{v}_t\left(v_h;r_s,n_2\right)-r_s}{v_h-r_h}=\frac{\int_{n_2/\alpha_s^t(\hat{v}_t(v_h;r_s,n_2);r_s)}^{+\infty}\min\left(1,k\left(\left(\gamma_2\alpha_h^t\left(\hat{v}_t\left(v_h;r_s,n_2\right);r_s\right)+\bar{\gamma}_2\alpha_h^\mu(r_s)\right)x\right)^{-1}\right)\mathrm{d}G\left(x\right)}{\int_{n_2/\alpha_s^t(\hat{v}_t(v_h;r_s,n_2);r_s)}^{+\infty}\min\left(1,k\left(\left(\gamma_2\alpha_s^t\left(\hat{v}_t\left(v_h;r_s,n_2\right);r_s\right)+\bar{\gamma}_2\alpha_s^\mu(r_s)\right)x\right)^{-1}\right)\mathrm{d}G\left(x\right)}$$

with $\bar{\gamma}_1=1-\gamma_1$ and $\bar{\gamma}_2=1-\gamma_2$. Let $\hat{v}_2\left(v_h\right)$ be the solution to the latter equation. Then $RHS\left(\hat{v}_2\left(v_h\right),r_s,n_2,\gamma_2\right)>RHS\left(\hat{v}_2\left(v_h\right),r_s,n_1,\gamma_1\right)$ because availability is lower on slow relative to hot when you have more strategic customers. Hence $LHS\left(\hat{v}_2\left(v_h\right),r_s\right)>RHS\left(\hat{v}_2\left(v_h\right),r_s,n_1,\gamma_1\right)$, which means that the solution to the former equation, $\hat{v}_1(v_h)$, will be lower than $\hat{v}_2(v_h)$, hence $LHS(\hat{v}_1\left(v_h\right),r_s)<LHS\left(\hat{v}_2\left(v_h\right),r_s\right)$, hence $RHS\left(\hat{v}_1\left(v_h\right),r_s,n_1,\gamma_1\right)<RHS\left(\hat{v}_2\left(v_h\right),r_s,n_2,\gamma_2\right)$, hence relative availability of hot vs slow must be higher in the latter equation, and since total visitors are the same because r_s is the same under both cases, this implies that $\alpha_s^t\left(r_s,n_1,\gamma_1\right)>\alpha_s^t\left(r_s,n_2,\gamma_2\right)$. □

Definition 1. Let (α_h,α_s) be the fractions of the population that, respectively, visit in the hot and in the slow period. We say that demand under $(\alpha_{h1},\alpha_{s1})$ is more smoothed than under $(\alpha_{h2},\alpha_{s2})$ if $\alpha_{h1}+\alpha_{s1}=\alpha_{h2}+\alpha_{s2}$ and $|\alpha_{h1}-\alpha_{s1}|<|\alpha_{h2}-\alpha_{s2}|$.

Let $\pi_p^\mu\left(x;r_s,\zeta\right)\triangleq r_h\min\left(k,\left(\left(1-\zeta\right)\alpha_h^\mu\left(r_s\right)+\zeta\bar{\alpha}_0\left(r_s\right)/2\right)x\right)+r_s\left(\left(1-\zeta\right)\alpha_s^\mu\left(r_s\right)+\zeta\bar{\alpha}_0\left(r_s\right)/2\right)x-2c_F$ be the profit of the firm when customers are non-strategic, the market size is x, the firm opens in the slow period charging a discounted price r_s, and the additional smoothing factor is equal to ζ, with $\alpha_s^\mu\left(r_s\right)=\int_0^{\bar{v}}\int_{r_s+(\tau_h-r_h)^+}^{\bar{v}}h\left(\tau_h,\tau_s\right)\mathrm{d}\tau_s\mathrm{d}\tau_h$, $\alpha_h^\mu\left(r_s\right)=\int_{r_h}^{\bar{v}}\int_0^{r_s+(\tau_h-r_h)^+}h\left(\tau_h,\tau_s\right)\mathrm{d}\tau_s\mathrm{d}\tau_h$, and with $\bar{\alpha}_0\left(r_s\right)=1-\alpha_0\left(r_s\right)$, where $\alpha_0\left(r_s\right)$ is defined as $\alpha_0\left(r_s\right)=\int_0^{r_h}\int_0^{r_s}h\left(\tau_h,\tau_s\right)\mathrm{d}\tau_s\mathrm{d}\tau_h$. When $\zeta=0$, $\pi_p^\mu\left(x;r_s,0\right)$ is equal to the profit of the firm under price discrimination, and under threshold discounting when the deal is active. Increasing ζ results in a progressively more smoothed demand, and for $\zeta=1$ demand is the same in both service periods. We consider the non-strategic case so that demand smoothing is fully and exogenously controlled by ζ.

Lemma 15. *Demand smoothing has diminishing returns on profit, that is,* $\int_0^{+\infty}\pi\left(x;r_s,\zeta\right)\mathrm{d}G\left(x\right)$ *is concave in* ζ.

Proof.

$$\frac{\partial}{\partial\zeta}\pi_p^{\mu}(x;r_s,\zeta)=r_h 1_{\left\{x<k/\left((1-\zeta)\alpha_h^{\mu}(r_s)+\zeta\bar{\alpha}_0(r_s)/2\right)\right\}}x\left(-\alpha_h^{\mu}(r_s)+\bar{\alpha}_0(r_s)/2\right)+r_s x\left(-\alpha_s^{\mu}(r_s)+\bar{\alpha}_0(r_s)/2\right)=$$

$$=r_h 1_{\left\{x<k/\left((1-\zeta)\alpha_h^{\mu}(r_s)+\zeta\bar{\alpha}_0(r_s)/2\right)\right\}}x\left(-\alpha_h^{\mu}(r_s)+\alpha_s^{\mu}(r_s)\right)/2+r_s x\left(-\alpha_s^{\mu}(r_s)+\alpha_h^{\mu}(r_s)\right)=$$

$$=\left(r_h 1_{\left\{x<k/\left((1-\zeta)\alpha_h^{\mu}(r_s)+\zeta\bar{\alpha}_0(r_s)/2\right)\right\}}-r_s\right)x\left(-\alpha_h^{\mu}(r_s)+\alpha_s^{\mu}(r_s)\right).$$

Note that $\frac{\partial^2}{\partial\zeta^2}\pi_p^{\mu}(r_s,\zeta;x)=0$. However,

$$\frac{\partial^2}{\partial\zeta^2}\Pi_p^{\mu}(r_s,\zeta)=$$

$$\frac{\partial}{\partial\zeta}\int_0^{k/\left((1-\zeta)\alpha_h^{\mu}(r_s)+\zeta\bar{\alpha}_0(r_s)/2\right)}-[r_h-r_s]\left(\alpha_h^{\mu}(r_s)-\alpha_s^{\mu}(r_s)\right)x\mathrm{d}G(x)+\frac{\partial}{\partial\zeta}\int_{k/\left((1-\zeta)\alpha_h^{\mu}(r_s)+\zeta\bar{\alpha}_0(r_s)/2\right)}^{+\infty}[r_s]\left(\alpha_h^{\mu}(r_s)-\alpha_s^{\mu}(r_s)\right)x\mathrm{d}G(x)=$$

$$=\underbrace{\frac{\partial}{\partial\zeta}\left(k/\left((1-\zeta)\alpha_h^{\mu}(r_s)+\zeta\bar{\alpha}_0(r_s)/2\right)\right)}_{>0\,iff\,\alpha_h^{\mu}>\alpha_s^{\mu}}\left[\underbrace{-r_h\left(\alpha_h^{\mu}(r_s)-\alpha_s^{\mu}(r_s)\right)}_{<0\,iff\,\alpha_h^{\mu}>\alpha_s^{\mu}}\right]\cdot$$

$$\cdot\underbrace{k/\left((1-\zeta)\alpha_h^{\mu}(r_s)+\zeta\bar{\alpha}_0(r_s)/2\right)G\left(k/\left((1-\zeta)\alpha_h^{\mu}(r_s)+\zeta\bar{\alpha}_0(r_s)/2\right)\right)}_{>0}<0$$

$\square$

Appendix D. Summary Of Notation

Notation	***Description***
$\mathbf{d}$	vector representing the deal announced by the firm. This vector includes the slow period price for all approaches, and also includes the activation threshold for the approach t.
$\Pi_j^\gamma(\mathbf{d})$	expected firm profit of approach j when the deal vector announced by the firm is $\mathbf{d}$ and the fraction of strategic customers is $\gamma \in [0,1]$.
$\pi_j^\gamma(\mathbf{d},x)$	ex-post profit of approach j when the deal vector announced by the firm is $\mathbf{d}$, the fraction of strategic customers is $\gamma \in [0,1]$, and the realized market size is x.
$\pi_{t-on}^\gamma(\mathbf{d},x)$	ex-post profit of approach t when the threshold is reached, the deal vector announced by the firm is $\mathbf{d}$, the fraction of strategic customers is $\gamma \in [0,1]$, and the realized market size is x.
$\breve{\pi}_j^\gamma(\mathbf{d},x)$	ex-post *variable* profit of approach j when the deal vector announced by the firm is $\mathbf{d}$, the fraction of strategic customers is $\gamma \in [0,1]$, and the realized market size is x.
r_j^γ	equilibrium price charged by the firm in the slow period under approach j.
$\alpha_i^j(\mathbf{d};\gamma)$	fraction of the market that visits in period $i \in \{h,s,0\}$ under approach j when the deal vector announced by the firm is $\mathbf{d}$ and the fraction of strategic customers in the population is γ.
$\hat{v}_j$	equilibrium slow period discount under approach j, where $\theta_j = 1 - r_s^j/r_h$
θ_j	equilibrium slow period valuation for the marginal customer under approach j

Note:1 $j \in \{c,p,t\}$ is a subscript that identifies the approach used (closure, price discrimination, threshold discounting)

Note2: the symbol γ may be omitted for brevity. The case of non-strategic customers ($\gamma = 0$) deserves special attention and is always identified by replacing γ with μ, e.g. $\Pi_j^\mu(\mathbf{d})$, $\alpha_i^j(\mathbf{d};\mu)$etc.

Note3: when a profit symbol appears without the deal vector $\mathbf{d}$, it refers to the equilibrium announcement vector, i.e. Π_j^γ, $\pi_{t-on}^\gamma(x)$ refer to the equilibrium profit.